Eur. Phys. J. A **21**, 11–28 (2004)
DOI 10.1140/epja/i2003-10178-y

THE EUROPEAN
PHYSICAL JOURNAL A

On kaonic hydrogen. Quantum field theoretic and relativistic covariant approach

A.N. Ivanov[a], M. Cargnelli[b], M. Faber[c], J. Marton[d], N.I. Troitskaya[e], and J. Zmeskal[f]

Atominstitut der Österreichischen Universitäten, Arbeitsbereich Kernphysik und Nukleare Astrophysik, Technische Universität Wien, Wiedner Hauptstr. 8-10, A-1040 Wien, Austria und Institut für Mittelenergiephysik, Österreichische Akademie der Wissenschaften, Boltzmanngasse 3, A-1090, Wien, Austria

Received: 7 October 2003 / Revised version: 29 November 2003 /
Published online: 22 June 2004 –
Communicated by V.V. Anisovich

Abstract. We study kaonic hydrogen, the bound K^-p state A_{Kp}. Within a quantum field theoretic and relativistic covariant approach we derive the energy level displacement of the ground state of kaonic hydrogen in terms of the amplitude of K^-p scattering for arbitrary relative momenta. The amplitude of low-energy K^-p scattering near threshold is defined by the contributions of three resonances $\Lambda(1405)$, $\Lambda(1800)$ and $\Sigma^0(1750)$ and a smooth elastic background. The amplitudes of inelastic channels of low-energy K^-p scattering fit experimental data on the near-threshold behaviour of the cross-sections and the experimental data by the DEAR Collaboration. We use the soft-pion technique (leading order in Chiral Perturbation Theory) for the calculation of the partial width of the radiative decay of pionic hydrogen $A_{\pi p} \to n + \gamma$ and the Panofsky ratio. The theoretical prediction for the Panofsky ratio agrees well with experimental data. We apply the soft-kaon technique (leading order in Chiral Perturbation Theory) to the calculation of the partial widths of radiative decays of kaonic hydrogen $A_{Kp} \to \Lambda^0 + \gamma$ and $A_{Kp} \to \Sigma^0 + \gamma$. We show that the contribution of these decays to the width of the energy level of the ground state of kaonic hydrogen is less than 1%.

PACS. 11.10.Ef Lagrangian and Hamiltonian approach – 13.75.Gx Pion-baryon interactions – 21.10.-k Properties of nuclei; nuclear energy levels – 36.10.-k Exotic atoms and molecules (containing mesons, muons, and other unusual particles)

1 Introduction

Kaonic hydrogen A_{Kp} is an analogy of hydrogen with an electron replaced by the K^--meson. The relative stability of kaonic hydrogen is fully due to Coulomb forces [1–8]. The Bohr radius of kaonic hydrogen is

$$a_{\rm B} = \frac{1}{\mu\,\alpha} = \frac{1}{\alpha}\Big(\frac{1}{m_{K^-}} + \frac{1}{m_p}\Big) = 83.594\,{\rm fm}, \qquad (1.1)$$

where $\mu = m_{K^-}m_p/(m_{K^-} + m_p) = 323.478\,{\rm MeV}$ is a reduced mass of the K^-p system, calculated at $m_{K^-} = 493.677\,{\rm MeV}$ and $m_p = 938.272\,{\rm MeV}$ [9], and $\alpha = e^2/\hbar c = 1/137.036$ is the fine-structure constant [9]. Below we use the units $\hbar = c = 1$, then $\alpha = e^2 = 1/137.036$. Since the Bohr radius of kaonic hydrogen is much greater than the range of strong low-energy interactions $R_{\rm str} \sim 1/m_{\pi^-} = 1.414\,{\rm fm}$, the strong low-energy interactions can be taken into account perturbatively [1–8].

According to Deser, Goldberger, Baumann and Thirring [1] the energy level displacement of the ground state of kaonic hydrogen can be defined in terms of the S-wave amplitude $f_0^{K^-p}(Q)$ of low-energy K^-p scattering as follows:

$$-\epsilon_{1s} + i\,\frac{\Gamma_{1s}}{2} = \frac{2\pi}{\mu}\,f_0^{K^-p}(0)\,|\Psi_{1s}(0)|^2, \qquad (1.2)$$

where $\Psi_{1s}(0) = 1/\sqrt{\pi a^3_{\rm B}}$ is the wave function of the ground state of kaonic hydrogen at the origin and $f_0^{K^-p}(0)$ is the amplitude of K^-p scattering in the S-wave state, calculated at zero relative momentum $Q = 0$ of the K^-p pair. The DGBT formula can be rewritten in the

[a] *Permanent address*: State Polytechnical University, Department of Nuclear Physics, 195251 St. Petersburg, Russian Federation; e-mail: ivanov@kph.tuwien.ac.at
[b] e-mail: michael.cargnelli@oeaw.ac.at
[c] e-mail: faber@kph.tuwien.ac.at
[d] e-mail: johann.marton@oeaw.ac.at
[e] *Permanent address*: State Polytechnical University, Department of Nuclear Physics, 195251 St. Petersburg, Russian Federation.
[f] e-mail: johann.zmeskal@oeaw.ac.at

equivalent form

$$-\epsilon_{1s} + i\,\frac{\Gamma_{1s}}{2} = 2\,\alpha^3\mu^2\, f_0^{K^-p}(0), \qquad (1.3)$$

where $2\,\alpha^3\mu^2 = 412.124\,\mathrm{eV\,fm^{-1}}$ and $f_0^{K^-p}(0)$ is measured in fm. The formula (1.3) is used by experimentalists for the analysis of experimental data on the energy level displacement of the ground state of kaonic hydrogen [10–13].

For non-zero relative momentum Q the amplitude $f_0^{K^-p}(Q)$ is defined by

$$f_0^{K^-p}(Q) = \frac{1}{2iQ}\Big(\eta_0^{K^-p}(Q)\, e^{2i\delta_0^{K^-p}(Q)} - 1\Big), \qquad (1.4)$$

where $\eta_0^{K^-p}(Q)$ and $\delta_0^{K^-p}(Q)$ are the inelasticity and the phase shift of the reaction $K^- + p \to K^- + p$, respectively. At relative momentum zero, $Q = 0$, the inelasticity and the phase shift are equal to $\eta_0^{K^-p}(0) = 1$ and $\delta_0^{K^-p}(0) = 0$. For $Q \to 0$ the phase shift behaves as $\delta_0^{K^-p}(Q) = a_0^{K^-p}\,Q + O(Q^2)$, where $a_0^{K^-p}$ is the S-wave scattering length of K^-p scattering.

The real part of $f_0^{K^-p}(0)$ is related to $a_0^{K^-p}$ as

$$\mathcal{R}e\, f_0^{K^-p}(0) = a_0^{K^-p} = \frac{1}{2}\,(a_0^0 + a_0^1), \qquad (1.5)$$

where a_0^0 and a_0^1 are the S-wave scattering lengths a_0^I with isospin $I = 0$ and $I = 1$, respectively.

Due to the optical theorem the imaginary part of the amplitude $f_0^{K^-p}(0)$ is related to the total cross-section $\sigma_0^{K^-p}(Q)$ for K^-p scattering in the S-wave state:

$$\mathcal{I}m\, f_0^{K^-p}(0) = \lim_{Q\to 0}\frac{Q}{4\pi}\,\sigma_0^{K^-p}(Q)$$
$$= \frac{1}{2}\lim_{Q\to 0}\frac{1}{Q}\,(1 - \eta_0^{K^-p}(Q)\cos 2\delta_0^{K^-p}(Q)). \qquad (1.6)$$

The r.h.s. of (1.6) can be transcribed into the form

$$\mathcal{I}m\, f_0^{K^-p}(0) = -\frac{1}{2}\,\frac{\mathrm{d}\eta_0^{K^-p}(Q)}{\mathrm{d}Q}\Big|_{Q=0}. \qquad (1.7)$$

Hence, according to the DGBT formula the energy level displacement of the ground state of kaonic hydrogen is defined by

$$\epsilon_{1s} = -2\,\alpha^3\mu^2\,\mathcal{R}e\, f_0^{K^-p}(0) = -2\,\alpha^3\mu^2\, a_0^{K^-p},$$
$$\Gamma_{1s} = 4\,\alpha^3\mu^2\,\mathcal{I}m\, f_0^{K^-p}(0) = -2\,\alpha^3\mu^2\,\frac{\mathrm{d}\eta_0^{K^-p}(Q)}{\mathrm{d}Q}\Big|_{Q=0}. \qquad (1.8)$$

The recent preliminary experimental data on the energy level displacement of the ground state of kaonic hydrogen obtained by the DEAR Collaboration [13] read

$$-\epsilon_{1s}^{\exp} + i\,\frac{\Gamma_{1s}^{\exp}}{2} = (-183 \pm 62) + i\,(106 \pm 69)\ \mathrm{eV}. \qquad (1.9)$$

In this paper we give i) a model-independent, quantum field theoretic and relativistic covariant derivation of the energy level displacement of the ground state of kaonic hydrogen and ii) a theoretical modeling of the amplitude of K^-p scattering in the S-wave state $f_0^{K^-p}(Q)$ near threshold of the K^-p pair $Q \approx 0$, fitting well experimental data (1.9) by the DEAR Collaboration [13].

The paper is organized as follows. In sect. 2 we write down the wave function of the ground state of kaonic hydrogen within the quantum field theoretic and relativistic covariant approach developed in [7,8] (see also [14]). In sect. 3 we derive the energy level displacement of the ground state of kaonic hydrogen in a model-independent way. In sect. 4 we describe the amplitude of K^-p scattering near threshold by the contributions of the resonances $\Lambda(1405)$, $\Lambda(1800)$ and $\Sigma(1750)$. The obtained amplitude of K^-p scattering we use for the calculation of the energy level displacement of the ground state of kaonic hydrogen. In sect. 5 we calculate the contribution of the elastic background to the amplitude of low-energy K^-p scattering. We show that the theoretical results fit well preliminary experimental data by the DEAR Collaboration [13]. In sect. 6 we calculate the partial widths of the radiative decay channels of kaonic hydrogen $A_{Kp} \to \Lambda^0 + \gamma$ and $A_{Kp} \to \Sigma^0 + \gamma$ [15]. First, we develop technique and methodics, based on the soft-pion(kaon) technique, for the calculation of the partial width of the decay $A_{\pi p} \to n + \gamma$ of pionic hydrogen in the ground state. We calculate the Panofsky ratio, $1/P = \Gamma(A_{\pi p} \to n + \gamma)/\Gamma((A_{\pi p} \to n + \pi^0) = 0.681 \pm 0.048$, in agreement with the experimental value $1/P = 0.647 \pm 0.004$ [16]. The application of this technique to the calculation of the partial widths of the decays $A_{Kp} \to \Lambda^0 + \gamma$ and $A_{Kp} \to \Sigma^0 + \gamma$ shows that the contribution of these decay channels to the width of the energy level of the ground state of kaonic hydrogen is less than 1%. In the conclusion we discuss the obtained results. We show that our approach to the description of low-energy K^-p scattering is consistent with the experimental data by the DEAR Collaboration [13]. In the appendix we calculate the elastic background of the S-wave elastic K^-p scattering near threshold within the Effective quark model with chiral $U(3) \times U(3)$ symmetry [17–19].

2 Ground-state wave function of kaonic hydrogen

The wave function of kaonic hydrogen in the ground state we define as [7,8,20,21]

$$|A_{Kp}^{(1s)}(\vec{P},\sigma_p)\rangle = \frac{1}{(2\pi)^3}\int\frac{\mathrm{d}^3k_{K^-}}{\sqrt{2E_{K^-}(\vec{k}_{K^-})}}\,\frac{\mathrm{d}^3k_p}{\sqrt{2E_p(\vec{k}_p)}}$$
$$\times\,\delta^{(3)}(\vec{P} - \vec{k}_{K^-} - \vec{k}_p)\sqrt{2E_A^{(1s)}(\vec{k}_{K^-} + \vec{k}_p)}$$
$$\times\,\Phi_{1s}(\vec{k}_{K^-})|K^-(\vec{k}_{K^-})p(\vec{k}_p,\sigma_p)\rangle, \qquad (2.1)$$

where $E_A^{(1s)}(\vec{P}\,) = \sqrt{M_A^{(1s)2} + \vec{P}^{\,2}}$ and $\vec{P}$ are total energy and momentum of kaonic hydrogen, respectively,

$M^{(1s)}_A = m_p + m_{K^-} + E_{1s}$ and $E_{1s} = -8613\,\mathrm{eV}$ are mass and binding energy of kaonic hydrogen in the ground bound state, respectively, σ_p is a polarization of the proton. Then, $\Phi_{1s}(\vec{k}_{K^-})$ is the wave function of the ground state in the momentum representation normalized by

$$\int \frac{d^3k}{(2\pi)^3}\,|\Phi_{1s}(\vec{k}\,)|^2 = 1. \qquad (2.2)$$

The wave function $|K^-(\vec{k}_{K^-})p(\vec{k}_p,\sigma_p)\rangle$ we define as [7,8, 20,21]

$$|K^-(\vec{k}_{K^-})p(\vec{k}_p,\sigma_p)\rangle = c^{\dagger}_{K^-}(\vec{k}_{K^-})a^{\dagger}_p(\vec{k}_p,\sigma_p)|0\rangle, \qquad (2.3)$$

where $c^{\dagger}_{K^-}(\vec{k}_{K^-})$ and $a^{\dagger}_p(\vec{k}_p,\sigma_p)$ are operators of creation of the K^--meson with momentum $\vec{k}_{K^-}$ and the proton with momentum $\vec{k}_p$ and polarization $\sigma_p = \pm 1/2$. They satisfy standard relativistic covariant commutation and anticommutation relations [7,20]. The wave function (2.1) is normalized by

$$\begin{aligned}&\langle A^{(1s)}_{Kp}(\vec{P}\,',\sigma\,'_p)|A^{(1s)}_{Kp}(\vec{P},\sigma_p)\rangle = \\ &(2\pi)^3\,2E^{(1s)}_A(\vec{P}\,)\,\delta^{(3)}(\vec{P}\,'-\vec{P}\,)\,\delta_{\sigma\,'_p\sigma_p}\int\frac{d^3k}{(2\pi)^3}\,|\Phi_{1s}(\vec{k}\,|^2 = \\ &(2\pi)^3\,2E^{(1s)}_A(\vec{P}\,)\,\delta^{(3)}(\vec{P}\,'-\vec{P}\,)\,\delta_{\sigma\,'_p\sigma_p}. \end{aligned}\qquad (2.4)$$

This is a relativistic covariant normalization of the wave function.

The wave function (2.1) we will apply to the calculation of the energy level displacement of the ground state of kaonic hydrogen within a quantum field theoretic and relativistic covariant approach.

3 Energy level displacement of the ground state

According to [7,8,20], the energy level displacement of the ground state of kaonic hydrogen is defined by

$$-\,\epsilon_{1s} + i\,\frac{\Gamma_{1s}}{2} = \lim_{T,V\to\infty}\frac{\langle A^{(1s)}_{Kp}(\vec{P},\sigma_p)|\mathbb{T}|A^{(1s)}_{Kp}(\vec{P},\sigma_p)\rangle}{2E^{(1s)}_A(\vec{P}\,)VT}\Big|_{\vec{P}=0}, \qquad (3.1)$$

where TV is a 4-dimensional volume defined by $(2\pi)^4\delta^{(4)}(0) = TV$ [20] and $\mathbb{T}$ is the T-matrix obeying the unitary condition [20,21]

$$\mathbb{T} - \mathbb{T}^{\dagger} = i\,\mathbb{T}^{\dagger}\mathbb{T}. \qquad (3.2)$$

Using the wave function (2.1) we reduce the r.h.s. of (3.1) to the form

$$\begin{aligned}-\,\epsilon_{1s} + i\,\frac{\Gamma_{1s}}{2} &= \frac{1}{4m_{K^-}m_p}\int\frac{d^3k}{(2\pi)^3}\int\frac{d^3q}{(2\pi)^3} \\ &\times\sqrt{\frac{m_{K^-}m_p}{E_{K^-}(\vec{k}\,)E_p(\vec{k}\,)}}\sqrt{\frac{m_{K^-}m_p}{E_{K^-}(\vec{q}\,)E_p(\vec{q}\,)}}\Phi^{\dagger}_{1s}(\vec{k}\,) \\ &\times\lim_{T,V\to\infty}\frac{\langle K^-(\vec{k}\,)p(-\vec{k},\sigma_p)|\mathbb{T}|K^-(\vec{q}\,)p(-\vec{q},\sigma_p)\rangle}{VT}\,\Phi_{1s}(\vec{q}\,),\end{aligned} \qquad (3.3)$$

where the matrix element of the T-matrix defines the amplitude of K^-p scattering[1]

$$\begin{aligned}&\lim_{T,V\to\infty}\frac{\langle K^-(\vec{k}\,)p(-\vec{k},\sigma_p)|\mathbb{T}|K^-(\vec{q}\,)p(-\vec{q},\sigma_p)\rangle}{VT} = \\ &M(K^-(\vec{q}\,)p(-\vec{q},\sigma_p)\to K^-(\vec{k}\,)p(-\vec{k},\sigma_p)).\end{aligned} \qquad (3.4)$$

Thus, the energy level displacement of the ground state of kaonic hydrogen is defined by the amplitude of K^-p scattering [7,8]:

$$\begin{aligned}-\epsilon_{1s} + i\,\frac{\Gamma_{1s}}{2} &= \frac{1}{4m_{K^-}m_p}\int\frac{d^3k}{(2\pi)^3}\int\frac{d^3q}{(2\pi)^3} \\ &\times\sqrt{\frac{m_{K^-}m_p}{E_{K^-}(\vec{k}\,)E_p(\vec{k}\,)}}\sqrt{\frac{m_{K^-}m_p}{E_{K^-}(\vec{q}\,)E_p(\vec{q}\,)}}\Phi^{\dagger}_{1s}(\vec{k}\,) \\ &\times M(K^-(\vec{q}\,)p(-\vec{q},\sigma_p)\to K^-(\vec{k}\,)p(-\vec{k},\sigma_p))\,\Phi_{1s}(\vec{q}\,),\end{aligned} \qquad (3.5)$$

Due to the wave functions $\Phi^{\dagger}_{1s}(\vec{k}\,)$ and $\Phi_{1s}(\vec{q}\,)$ the main contributions to the integrals over $\vec{k}$ and $\vec{q}$ come from the regions of 3-momenta $k \sim 1/a_{\rm B}$ and $q \sim 1/a_{\rm B}$, where $1/a_{\rm B} = 2.361\,\mathrm{MeV}$. Since typical momenta in the integrand are much less than the masses of coupled particles, $m_{K^-} \gg 1/a_{\rm B}$ and $m_p \gg 1/a_{\rm B}$, the amplitude of K^-p scattering can be defined for low-energy momenta only[2].

Following [7,8] the amplitude of low-energy K^-p scattering we define as

$$\begin{aligned}&M(K^-(\vec{q}\,)p(-\vec{q},\sigma_p)\to K^-(\vec{k}\,)p(-\vec{k},\sigma_p)) = \\ &8\pi\,(m_{K^-}+m_p)\,f^{K^-p}_0(\sqrt{kq}),\end{aligned} \qquad (3.6)$$

where the amplitude $f^{K^-p}_0(\sqrt{kq})$ is determined by

$$f^{K^-p}_0(\sqrt{kq}) = \frac{1}{2i\sqrt{kq}}\Big(\eta^{K^-p}_0(\sqrt{kq})\,e^{2i\delta^{K^-p}_0(\sqrt{kq})} - 1\Big). \qquad (3.7)$$

[1] In Chiral Perturbation Theory (ChPT) [22,23] the T-matrix can be expressed in terms of an effective Lagrangian $\mathcal{L}_{\rm eff}(x)$ (see also [7,8]). If all loop contributions are taken into account and renormalization is carried out, the effective Lagrangian $\mathcal{L}_{\rm eff}(x)$ can be used only in the tree-approximation [24] (see also [7,8]).

[2] It is obvious that due to formula (3.5) a knowledge of the amplitude of K^-p scattering for all relative momenta from zero to infinity should give a possibility to calculate the energy level displacement of the ground state of kaonic hydrogen without any low-energy approximation.

The shift and width of the energy level of the ground state of kaonic hydrogen are equal to

$$\epsilon_{1s} = -\frac{2\pi}{\mu}\iint \frac{d^3k}{(2\pi)^3}\frac{d^3q}{(2\pi)^3}\sqrt{\frac{m_{K^-}m_p}{E_{K^-}(\vec{k}\,)E_p(\vec{k}\,)}} \times\sqrt{\frac{m_{K^-}m_p}{E_{K^-}(\vec{q}\,)E_p(\vec{q}\,)}}\Phi^{\dagger}_{1s}(\vec{k}\,)\Phi_{1s}(\vec{q}\,) \times \eta_0^{K^-p}(\sqrt{kq})\,\frac{\sin 2\delta_0^{K^-p}(\sqrt{kq})}{2\sqrt{kq}},$$

$$\Gamma_{1s} = \frac{2\pi}{\mu}\iint \frac{d^3k}{(2\pi)^3}\frac{d^3q}{(2\pi)^3}\sqrt{\frac{m_{K^-}m_p}{E_{K^-}(\vec{k}\,)E_p(\vec{k}\,)}} \times\sqrt{\frac{m_{K^-}m_p}{E_{K^-}(\vec{q}\,)E_p(\vec{q}\,)}}\,\Phi^{\dagger}_{1s}(\vec{k}\,)\Phi_{1s}(\vec{q}\,) \times\frac{1}{\sqrt{kq}}\,(1-\eta_0^{K^-p}(\sqrt{kq})\,\cos 2\delta_0^{K^-p}(\sqrt{kq})) = \frac{1}{2\mu}\iint \frac{d^3k}{(2\pi)^3}\frac{d^3q}{(2\pi)^3}\sqrt{\frac{m_{K^-}m_p}{E_{K^-}(\vec{k}\,)E_p(\vec{k}\,)}} \times\sqrt{\frac{m_{K^-}m_p}{E_{K^-}(\vec{q}\,)E_p(\vec{q}\,)}}\,\Phi^{\dagger}_{1s}(\vec{k}\,)\Phi_{1s}(\vec{q}\,) \times\sqrt{kq}\,\sigma_0^{K^-p}(\sqrt{kq}). \quad (3.8)$$

Formula (3.8) reduces to the DGBT formula defining the amplitude of K^-p scattering at $k = q = 0$ [7,8]. We would like to emphasize that the main contributions to the momentum integrals in (3.8) come from the region $k \sim q \sim 1/a_{\rm B} = 2.361\,{\rm MeV}$ but not from $k = q = 0$. Hence, the calculation of the amplitude of K^-p scattering at $k = q = 0$ is not an explicit result but an approximation, which is well-defined only if the amplitude of K^-p scattering is a smooth function near threshold[3].

Assuming that near threshold the amplitude of low-energy K^-p scattering is a smooth function of relative momentum Q of the K^-p pair and keeping only the leading terms in momentum expansion at $Q = 0$, we arrive at the energy level displacement of the ground state of kaonic hydrogen:

$$-\epsilon_{1s} + i\,\frac{\Gamma_{1s}}{2} = \frac{2\pi}{\mu}\left[a_0^{K^-p} - i\,\frac{1}{2}\,\frac{d\eta_0^{K^-p}(Q)}{dQ}\Big|_{Q=0}\right] \times\left|\int\frac{d^3k}{(2\pi)^3}\sqrt{\frac{m_{K^-}m_p}{E_{K^-}(\vec{k}\,)E_p(\vec{k}\,)}}\,\Phi_{1s}(\vec{k}\,)\right|^2. \quad (3.9)$$

[3] Practically, the corrections to the energy level displacement, coming from a momentum expansion of the amplitude of K^-p scattering, are of order of powers of α. This means that the term of order $O(Q)$ gives a correction of order $O(\alpha)$, multiplied by the derivative of the amplitude of K^-p scattering with respect to the relative momentum Q, calculated at $Q = 0$. The convergence of this expansion is fully defined by the derivatives of the amplitude of K^-p scattering. Such corrections, caused by Coulombic photons, should be taken into account on the same footing as the corrections caused by QCD isospin-breaking and electromagnetic interactions [25,26] (see also [27]).

This is the quantum field theoretic, relativistic covariant and model-independent generalization of the DGBT formula (1.2) [7,8].

The amplitude of low-energy K^-p scattering we represent in the form

$$f_0^{K^-p}(Q) = \frac{1}{2iQ}\left(\eta_0^{K^-p}(Q)\,e^{2i\delta_0^{K^-p}(Q)} - 1\right) = \frac{1}{2iQ}\left(e^{2i\delta_B^{K^-p}(Q)} - 1\right) + e^{2i\delta_B^{K^-p}(Q)}f_0^{K^-p}(Q)_R, \quad (3.10)$$

where $\delta_0^{K^-p}(Q)_B$ is the phase shift of an elastic background of low-energy K^-p scattering and $f_0^{K^-p}(Q)_R$ is the contribution of resonances.

We assume that $f_0^{K^-p}(Q)_R$ is defined by the contributions of the $\Lambda(1405)$-resonance, an $SU(3)_{\rm flavour}$ singlet [28], and the $\Lambda(1800)$ and $\Sigma(1750)$ resonances, components of the $SU(3)_{\rm flavour}$ octet [29] [4]. For simplicity we denote $\Lambda(1405)$ as Λ^0_1 and $\Lambda(1800)$ and $\Sigma(1750)$ as Λ^0_2 and Σ^0_2 [5], respectively.

4 Amplitude of low-energy K$^-$p scattering. Resonances

Treating the resonances $\Lambda(1405)$, $\Lambda(1800)$ and $\Sigma(1750)$ as elementary fields[6] we can write down phenomenological interactions:

$$\mathcal{L}_{\Lambda_1BP}(x) = g_1\bar{\Lambda}^0_1(x)\,{\rm tr}\{B(x)P(x)\} + {\rm h.c.} = g_1\bar{\Lambda}_1(x)\,B^b_a(x)P^a_b(x) + {\rm h.c.},$$

$$\mathcal{L}_{B_2BP}(x) = \frac{1}{\sqrt{2}}\,g_2\,{\rm tr}\{\{\bar{B}_2, B\}P\} + \frac{1}{\sqrt{2}}\,f_2\,{\rm tr}\{[\bar{B}_2, B]P\} + {\rm h.c.} = \frac{1}{\sqrt{2}}\,(g_2+f_2)\,(\bar{B}_2)^b_aB^a_cP^c_b + \frac{1}{\sqrt{2}}\,(g_2-f_2)\,(\bar{B}_2)^b_aB^c_bP^a_c + {\rm h.c.}, \quad (4.1)$$

where g_1, g_2 and f_2 are phenomenological coupling constants, $\Lambda^0_1(x)$, $(\bar{B}_2)^b_a(x)$, $B^b_a(x)$ and $P^a_b(x)\,(a(b) = 1,2,\ldots,8)$ are interpolating fields of the $\Lambda(1405)$-resonance, the octet of baryon resonances $\Lambda(1800)$ and $\Sigma(1750)$, the octet of light baryons and the octet of

[4] Recall that the resonance $\Lambda(1405)$ has a status $****$, whereas the resonances $\Lambda(1800)$ and $\Sigma(1750)$ have a status $***$ [28,29].

[5] We keep only the neutral component of the $\Sigma(1750)$-resonance.

[6] This agrees, for instance, with the approach developed within ChPT in [30].

pseudoscalar mesons, respectively:

$$(\bar{B}_2)^b_a = \begin{pmatrix} \dfrac{\bar{\Sigma}^0_2}{\sqrt{2}} + \dfrac{\bar{\Lambda}^0_2}{\sqrt{6}} & \bar{\Sigma}^-_2 & -\bar{\Xi}^-_2 \\ \bar{\Sigma}^+_2 & -\dfrac{\bar{\Sigma}^0_2}{\sqrt{2}} + \dfrac{\bar{\Lambda}^0_2}{\sqrt{6}} & \bar{\Xi}^0_2 \\ \bar{p}_2 & \bar{n}_2 & -\dfrac{2}{\sqrt{6}}\bar{\Lambda}^0_2 \end{pmatrix},$$

$$B^b_a = \begin{pmatrix} \dfrac{\Sigma^0}{\sqrt{2}} + \dfrac{\Lambda^0}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\dfrac{\Sigma^0}{\sqrt{2}} + \dfrac{\Lambda^0}{\sqrt{6}} & n \\ -\Xi^- & \Xi^0 & -\dfrac{2}{\sqrt{6}}\Lambda^0 \end{pmatrix},$$

$$P^a_b = \begin{pmatrix} \dfrac{\pi^0}{\sqrt{2}} + \dfrac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\dfrac{\pi^0}{\sqrt{2}} + \dfrac{\eta}{\sqrt{6}} & K^0 \\ -K^- & \bar{K}^0 & -\dfrac{2}{\sqrt{6}}\,\eta \end{pmatrix}. \quad (4.2)$$

For simplicity we identify the component $\eta(x)$ of the pseudoscalar octet with the observed pseudoscalar meson $\eta(550)$ [9].

Keeping only terms relevant to low-energy K^-p scattering we reduce the effective Lagrangians (4.1) to the form

$$\begin{aligned}
\mathcal{L}_{\Lambda^0_1 BP}(x) &= g_1\,\bar{\Lambda}^0_1(x)(\vec{\Sigma}(x)\cdot\vec{\pi}(x) - p(x)K^-(x) \\
&\quad + n(x)\bar{K}^0(x) + \frac{1}{3}\,\Lambda^0(x)\eta(x)) + \text{h.c.} \\
\mathcal{L}_{\Lambda^0_2 BP}(x) &= \frac{g_2}{\sqrt{3}}\,\bar{\Lambda}^0_2(x)(\vec{\Sigma}(x)\cdot\vec{\pi}(x) - \Lambda^0(x)\eta(x)) \\
&\quad + \frac{g_2 + 3f_2}{2\sqrt{3}}\,\bar{\Lambda}^0_2(x)\,(p(x)K^-(x) \\
&\quad - n(x)\bar{K}^0(x)) + \text{h.c.}, \\
\mathcal{L}_{\Sigma^0_2 BP}(x) &= f_2\,\bar{\Sigma}^0_2(x)\,(\Sigma^-(x)\pi^+(x) - \Sigma^+(x)\pi^-(x)) \\
&\quad + \frac{g_2}{\sqrt{3}}\,\bar{\Sigma}^0_2(x)\,(\Lambda^0(x)\pi^0(x) + \Sigma^0(x)\eta(x)) \\
&\quad + \frac{g_2 - f_2}{2}\,\bar{\Sigma}^0_2(x)\,(-p(x)K^-(x) \\
&\quad - n(x)\bar{K}^0(x)) + \text{h.c.}\,. \qquad (4.3)
\end{aligned}$$

According to (3.10) at threshold $Q = 0$ the amplitude $f^{K^-p}_0(0)$ of K^-p scattering we define as

$$f^{K^-p}_0(0) = A^{K^-p}_B + f^{K^-p}_0(0)_R, \quad (4.4)$$

where $A^{K^-p}_B$ is a real parameter[7], describing a smooth elastic background $\delta^{K^-p}_0(Q)_B = A^{K^-p}_B Q$, and $f^{K^-p}_0(0)_R$ is the contribution of the resonances, which we determine as

$$f^{K^-p}_0(0)_R = \frac{1}{2}\left(f^{K^-p}_0(0)_{I=0} + f^{K^-p}_0(0)_{I=1}\right), \quad (4.5)$$

[7] We calculate the parameter $A^{K^-p}_B$ in sect. 5.

where the amplitudes $f^{K^-p}_0(0)_{I=0}$ and $f^{K^-p}_0(0)_{I=1}$ of low-energy K^-p scattering with isospin $I = 0$ and isospin $I = 1$ are saturated by the $\Lambda(1405)$, $\Lambda(1800)$ and $\Sigma(1750)$ resonances, respectively. The amplitude $f^{K^-p}_0(0)_R$ contains real and imaginary parts $\mathcal{R}e\, f^{K^-p}_0(0)_R$ and $\mathcal{I}m\, f^{K^-p}_0(0)_R$, which define elastic and inelastic channels.

4.1 Imaginary part of $\mathbf{f^{K^-p}_0(0)_R}$

The imaginary part $\mathcal{I}m\, f^{K^-p}_0(0)_R$ of the amplitude $f^{K^-p}_0(0)_R$ is determined by inelastic channels. The near-threshold low-energy K^-p interaction contains four inelastic channels defined by strong low-energy interactions: i) $K^-p \to \Sigma^-\pi^+$, ii) $K^-p \to \Sigma^+\pi^-$, iii) $K^-p \to \Sigma^0\pi^0$ and iv) $K^-p \to \Lambda^0\pi^0$. The amplitudes of these channels we define as [30]

$$\begin{aligned}
f(K^-p \to \Sigma^-\pi^+) &= \frac{1}{4\pi}\,\frac{\mu}{m_{K^-}}\,\sqrt{\frac{m_{\Sigma^-}}{m_p}} \\
&\times\Bigg[-\frac{g^2_1}{m_{\Lambda^0_1} - m_{K^-} - m_p} + \frac{1}{6}\,\frac{g^2_2\,(1 + 3\alpha_2)}{m_{\Lambda^0_2} - m_{K^-} - m_p} \\
&\quad - \frac{1}{2}\,\frac{g^2_2\,\alpha_2\,(1 - \alpha_2)}{m_{\Sigma^0_2} - m_{K^-} - m_p}\Bigg], \\
f(K^-p \to \Sigma^+\pi^-) &= \frac{1}{4\pi}\,\frac{\mu}{m_{K^-}}\,\sqrt{\frac{m_{\Sigma^+}}{m_p}} \\
&\times\Bigg[-\frac{g^2_1}{m_{\Lambda^0_1} - m_{K^-} - m_p} + \frac{1}{6}\,\frac{g^2_2(1 + 3\alpha_2)}{m_{\Lambda^0_2} - m_{K^-} - m_p} \\
&\quad + \frac{1}{2}\,\frac{g^2_2\,\alpha_2\,(1 - \alpha_2)}{m_{\Sigma^0_2} - m_{K^-} - m_p}\Bigg], \\
f(K^-p \to \Sigma^0\pi^0) &= \frac{1}{4\pi}\,\frac{\mu}{m_{K^-}}\,\sqrt{\frac{m_{\Sigma^0}}{m_p}} \\
&\times\Bigg[-\frac{g^2_1}{m_{\Lambda^0_1} - m_{K^-} - m_p} + \frac{1}{6}\,\frac{g^2_2\,(1 + 3\alpha_2)}{m_{\Lambda^0_2} - m_{K^-} - m_p}\Bigg], \\
f(K^-p \to \Lambda^0\pi^0) &= \frac{1}{4\pi}\,\frac{\mu}{m_{K^-}}\,\sqrt{\frac{m_{\Lambda^0}}{m_p}} \\
&\times\Bigg[-\frac{1}{2}\,\frac{1}{\sqrt{3}}\,\frac{g^2_2\,(1 - \alpha_2)}{m_{\Sigma^0_2} - m_{K^-} - m_p}\Bigg], \qquad (4.6)
\end{aligned}$$

where $\alpha_2 = f_2/g_2$.

In order to check the consistency of our approach we suggest to use experimental data on the cross-sections for the inelastic reactions $K^-p \to \Sigma^-\pi^+$, $K^-p \to \Sigma^+\pi^-$, $K^-p \to \Sigma^0\pi^0$ and $K^-p \to \Lambda^0\pi^0$ taken at threshold of the K^-p pair [31,32]

see equation (4.7) *on the next page*

These data should place constraints on the input parameters of any approach [33]. In terms of the amplitudes of

$$\gamma = \frac{\sigma(K^-p \to \Sigma^-\pi^+)}{\sigma(K^-p \to \Sigma^+\pi^-)} = 2.360 \pm 0.040,$$
$$R_c = \frac{\sigma(K^-p \to \Sigma^-\pi^+) + \sigma(K^-p \to \Sigma^+\pi^-)}{\sigma(K^-p \to \Sigma^-\pi^+) + \sigma(K^-p \to \Sigma^+\pi^-) + \sigma(K^-p \to \Sigma^0\pi^0) + \sigma(K^-p \to \Lambda^0\pi^0)} = 0.664 \pm 0.011,$$
$$R_n = \frac{\sigma(K^-p \to \Lambda^0\pi^0)}{\sigma(K^-p \to \Sigma^0\pi^0) + \sigma(K^-p \to \Lambda^0\pi^0)} = 0.189 \pm 0.015. \quad (4.7)$$

the inelastic reactions under consideration they read

$$\gamma = \frac{|f(K^-p \to \Sigma^-\pi^+)|^2 k_{\Sigma^-\pi^+}}{|f(K^-p \to \Sigma^+\pi^-)|^2 k_{\Sigma^+\pi^-}},$$
$$R_c = \Big(|f(K^-p \to \Sigma^-\pi^+)|^2 k_{\Sigma^-\pi^+} + |f(K^-p \to \Sigma^+\pi^-)|^2 k_{\Sigma^+\pi^-}\Big) \times \Big(|f(K^-p \to \Sigma^-\pi^+)|^2 k_{\Sigma^-\pi^+} + |f(K^-p \to \Sigma^+\pi^-)|^2 k_{\Sigma^+\pi^-} + |f(K^-p \to \Sigma^0\pi^0)|^2 k_{\Sigma^0\pi^0} + |f(K^-p \to \Lambda^0\pi^0)|^2 k_{\Lambda^0\pi^0}\Big)^{-1},$$
$$R_n = \frac{|f(K^-p \to \Lambda^0\pi^0)|^2 k_{\Lambda^0\pi^0}}{|f(K^-p\to\Sigma^0\pi^0)|^2 k_{\Sigma^0\pi^0} + |f(K^-p\to\Lambda^0\pi^0)|^2 k_{\Lambda^0\pi^0}}, \quad (4.8)$$

where k_{AB} with $A = \Sigma^\pm, \Sigma^0, \Lambda^0$ and $B = \pi^\pm, \pi^0$ is a relative momentum of the AB pair, calculated at threshold

$$k_{AB}(s) = \frac{1}{2\sqrt{s}}\sqrt{(s-(m_A+m_B)^2)(s-(m_A-m_B)^2)}. \quad (4.9)$$

At threshold $s = (m_{K^-} + m_p)^2$ and $k_{AB}((m_{K^-} + m_p)^2) = k_{AB}$.

Expressing the amplitudes of inelastic channels with neutral particles in the final states in terms of the amplitudes of the reactions with charged particles in the final state we get

$$f(K^-p \to \Sigma^0\pi^0) = \frac{1}{2}\left[\sqrt{\frac{m_{\Sigma^0}}{m_{\Sigma^-}}}\, f(K^-p \to \Sigma^-\pi^+) + \sqrt{\frac{m_{\Sigma^0}}{m_{\Sigma^+}}}\, f(K^-p \to \Sigma^+\pi^-)\right],$$
$$f(K^-p \to \Lambda^0\pi^0) = \frac{1}{\alpha_2}\frac{1}{2\sqrt{3}}\left[\sqrt{\frac{m_{\Lambda^0}}{m_{\Sigma^-}}}\, f(K^-p \to \Sigma^-\pi^+) - \sqrt{\frac{m_{\Lambda^0}}{m_{\Sigma^+}}}\, f(K^-p \to \Sigma^+\pi^-)\right]. \quad (4.10)$$

Combining relations (4.10) and (4.8) we express the amplitudes of inelastic channels $K^-p \to \Sigma^+\pi^-$, $K^-p \to \Sigma^0\pi^0$ and $K^-p \to \Lambda^0\pi^0$ in terms of the amplitude of the reaction $K^-p \to \Sigma^-\pi^+$. This gives

$$f(K^-p \to \Sigma^+\pi^-) = f(K^-p \to \Sigma^-\pi^+)\sqrt{\frac{1}{\gamma}\,\frac{k_{\Sigma^-\pi^+}}{k_{\Sigma^+\pi^-}}},$$
$$f(K^-p \to \Sigma^0\pi^0) = f(K^-p \to \Sigma^-\pi^+)\frac{1}{2}\sqrt{\frac{m_{\Sigma^0}}{m_{\Sigma^-}}} \times \left(1 + \sqrt{\frac{1}{\gamma}\,\frac{m_{\Sigma^-}}{m_{\Sigma^+}}\,\frac{k_{\Sigma^-\pi^+}}{k_{\Sigma^+\pi^-}}}\right),$$
$$f(K^-p \to \Lambda^0\pi^0) = f(K^-p \to \Sigma^-\pi^+)\sqrt{\frac{R_n}{1-R_n}\,\frac{k_{\Sigma^0\pi^0}}{k_{\Lambda^0\pi^0}}} \times \frac{1}{2}\sqrt{\frac{m_{\Lambda^0}}{m_{\Sigma^-}}}\left(1 + \sqrt{\frac{1}{\gamma}\,\frac{m_{\Sigma^-}}{m_{\Sigma^+}}\,\frac{k_{\Sigma^-\pi^+}}{k_{\Sigma^+\pi^-}}}\right). \quad (4.11)$$

The parameter α_2 is defined by

$$\alpha_2 = -\sqrt{\frac{1-R_n}{3R_n}\,\frac{k_{\Lambda^0\pi^0}}{k_{\Sigma^0\pi^0}}}\;\frac{1 - \sqrt{\frac{1}{\gamma}\,\frac{m_{\Sigma^-}}{m_{\Sigma^+}}\,\frac{k_{\Sigma^-\pi^+}}{k_{\Sigma^+\pi^-}}}}{1 + \sqrt{\frac{1}{\gamma}\,\frac{m_{\Sigma^-}}{m_{\Sigma^+}}\,\frac{k_{\Sigma^-\pi^+}}{k_{\Sigma^+\pi^-}}}}. \quad (4.12)$$

In our approach the parameter R_c turns out to be dependent and reads

see equation (4.13) *on the next page*

Using the experimental values of γ, R_n and masses of baryons and mesons [9] we get

$$R_c = 0.626 \pm 0.007,$$
$$\alpha_2 = -0.314 \pm 0.026, \quad (4.14)$$

where uncertainties are caused by the experimental errors of the parameters γ and R_n.

Comparing the theoretical prediction $R_c = 0.626 \pm 0.007$ with the experimental value $R_c = 0.664 \pm 0.011$ in (4.7) we can argue that our approach to the description of K^-p scattering near threshold is consistent with experimental data on the cross-sections for the inelastic reactions within an accuracy better than 6%.

Hence, using the relations γ and R_c for the cross-sections for the inelastic reactions we can write down

$$\sigma(K^-p \to \text{all}) = \sum_X \sigma(K^-p \to X) = \frac{1}{R_c}\left(1 + \frac{1}{\gamma}\right)\sigma(K^-p \to \Sigma^-\pi^+), \quad (4.15)$$

$$R_c = \frac{1}{\displaystyle 1 + \frac{1}{4}\frac{\gamma}{\gamma+1}\frac{k_{\Sigma^0\pi^0}}{k_{\Sigma^-\pi^+}}\left(\frac{m_{\Sigma^0}}{m_{\Sigma^-}} + \frac{R_n}{1-R_n}\frac{m_{\Lambda^0}}{m_{\Sigma^-}}\right)\left(1+\sqrt{\frac{1}{\gamma}\frac{m_{\Sigma^-}}{m_{\Sigma^+}}\frac{k_{\Sigma^-\pi^+}}{k_{\Sigma^+\pi^-}}}\right)^2}. \qquad (4.13)$$

where $X = \Sigma^-\pi^+, \Sigma^+\pi^-, \Sigma^0\pi^0$ and $\Lambda^0\pi^0$.

Due to the optical theorem relation (4.15) determines the imaginary part of the amplitude $f_0^{K^-p}(0)_R$. It reads

$$\mathcal{I}m\, f_0^{K^-p}(0)_R = \frac{1}{R_c}\left(1+\frac{1}{\gamma}\right)|f(K^-p\to\Sigma^-\pi^+)|^2 k_{\Sigma^-\pi^+}. \qquad (4.16)$$

Since in our approach $\mathcal{I}m\, f_0^{K^-p}(0) = \mathcal{I}m\, f_0^{K^-p}(0)_R$, relation (4.16) allows to determine the total width of kaonic hydrogen Γ_{1s} in terms of the partial width of the decay $A_{Kp}\to\Sigma^- + \pi^+$ [33]

$$\Gamma_{1s} = \frac{1}{R_c}\left(1+\frac{1}{\gamma}\right)\Gamma(A_{Kp}\to\Sigma^-\pi^+) = 842.248\,\mathcal{I}m\, f_0^{K^-p}(0) = 842.248\,\frac{1}{R_c}\left(1+\frac{1}{\gamma}\right)|f(K^-p\to\Sigma^-\pi^+)|^2 k_{\Sigma^-\pi^+}\ \mathrm{eV}. \qquad (4.17)$$

For the calculation of the numerical value of $f(K^-p\to\Sigma^-\pi^+)$ we have to determine the coupling constant g_1 and g_2. They can be obtained fitting the total experimental widths of the resonances $\Lambda(1405)$, $\Lambda(1800)$ and $\Sigma(1750)$ [9]. We would like to remark that within an accuracy better than 6% we can set $\alpha_2 = -1/3$ and neglect the contribution of the $\Lambda(1800)$-resonance. Therefore, the constant g_2 we define from the experimental data on the $\Sigma(1750)$-resonance only.

We would like to emphasize that the experimental data on the masses and total widths of the $\Lambda(1405)$ and $\Sigma(1750)$ resonances are rather ambiguous. Below we use only recommended values for the masses and total widths of these resonances [9].

4.1.1 The $\Lambda(1405)$-resonance

The recommended values for the mass and total width of the $\Lambda(1405)$-resonance are equal to $m_{\Lambda^0_1} = 1406\,\mathrm{MeV}$ and $\Gamma_{\Lambda^0_1} = 50\,\mathrm{MeV}$ [28,34].

The total width of the $\Lambda(1405)$-resonance is defined by the decays $\Lambda(1405)\to\Sigma+\pi$ [9]. Due to the effective Lagrangian (4.3) the total width of the $\Lambda(1405)$-resonance $\Gamma_{\Lambda^0_1}$ reads

$$\Gamma_{\Lambda^0_1} = \frac{g_1^2}{8\pi}\frac{(m_{\Lambda^0_1}+m_{\Sigma^+})^2 - m^2_{\pi^-}}{m^2_{\Lambda^0_1}}k_{\Sigma^+\pi^-} + \frac{g_1^2}{8\pi}\frac{(m_{\Lambda^0_1}+m_{\Sigma^-})^2 - m^2_{\pi^+}}{m^2_{\Lambda^0_1}}k_{\Sigma^-\pi^+} + \frac{g_1^2}{8\pi}\frac{(m_{\Lambda^0_1}+m_{\Sigma^0})^2 - m^2_{\pi^0}}{m^2_{\Lambda^0_1}}k_{\Sigma^0\pi^0}. \qquad (4.18)$$

Setting $\Gamma_{\Lambda^0_1} = 50\,\mathrm{MeV}$ and using the experimental values for the masses of the Σ-hyperon and π-meson [9], we obtain the value of the coupling constant g_1: $g_1 = 0.907$.

4.1.2 The $\Sigma(1750)$-resonance

The recommended values for the mass and total width of the $\Sigma(1750)$-resonance are equal to $m_{\Sigma^0_2} = 1750\,\mathrm{MeV}$ and $\Gamma_{\Sigma^0_2} = 90\,\mathrm{MeV}$ [29,35]. From the Lagrangian (4.3) we define the total width of the $\Sigma(1750)$-resonance:

$$\Gamma_{\Sigma^0_2} = \frac{g_2^2}{72\pi}\frac{(m_{\Sigma^0_2}+m_{\Sigma^+})^2 - m^2_{\pi^-}}{m^2_{\Sigma^0_2}}k_{\Sigma^+\pi^-} + \frac{g_2^2}{72\pi}\frac{(m_{\Sigma^0_2}+m_{\Sigma^-})^2 - m^2_{\pi^+}}{m^2_{\Sigma^0_2}}k_{\Sigma^-\pi^+} + \frac{g_2^2}{24\pi}\frac{(m_{\Sigma^0_2}+m_{\Lambda^0})^2 - m^2_{\pi^0}}{m^2_{\Sigma^0_2}}k_{\Lambda^0\pi^0} + \frac{g_2^2}{24\pi}\frac{(m_{\Sigma^0_2}+m_{\Sigma^0})^2 - m^2_{\eta}}{m^2_{\Sigma^0_2}}k_{\Sigma^0\eta} + \frac{g_2^2}{18\pi}\frac{(m_{\Sigma^0_2}+m_p)^2 - m^2_{K^-}}{m^2_{\Sigma^0_2}}k_{pK^-} + \frac{g_2^2}{18\pi}\frac{(m_{\Sigma^0_2}+m_n)^2 - m^2_{\bar{K}^0}}{m^2_{\Sigma^0_2}}k_{n\bar{K}^0}, \qquad (4.19)$$

where we have used $\alpha_2 = -1/3$. Setting $\Gamma_{\Sigma^0_2} = 90\,\mathrm{MeV}$ and using experimental values for the masses of baryons and mesons we get $g_2 = 1.123$.

4.1.3 Numerical values of $f(K^-p\to\Sigma^-\pi^+)$ and imaginary part of $f_0^{K^-p}(0)_R$

Setting $\alpha_2 = -1/3$ in (4.6) and using the coupling constant $g_1 = 0.907$ and $g_2 = 1.123$, calculated above, we obtain the numerical value of the amplitude $f(K^-p \to \Sigma^-\pi^+)$

$$f(K^-p \to \Sigma^-\pi^+) = \frac{1}{4\pi}\,\frac{\mu}{m_{K^-}}\sqrt{\frac{m_{\Sigma^-}}{m_p}} \times \Big[-\frac{g_1^2}{m_{\Lambda_1^0} - m_{K^-} - m_p} + \frac{2}{9}\,\frac{g_2^2}{m_{\Sigma_2^0} - m_{K^-} - m_p}\Big] = (0.379 \pm 0.023)\,\mathrm{fm}\,. \qquad (4.20)$$

Due to relation (4.16) this gives the imaginary part of the amplitude $f_0^{K^-p}(0)_R$:

$$\mathcal{I}m\, f_0^{K^-p}(0)_R = (0.269 \pm 0.032)\ \mathrm{fm}. \qquad (4.21)$$

According to this value and the relation $\mathcal{I}m\, f_0^{K^-p}(0) = \mathcal{I}m\, f_0^{K^-p}(0)_R$, the total width Γ_{1s} of kaonic hydrogen in the ground state should be equal to

$$\Gamma_{1s}^{\rm th} = 842.248\,\mathcal{I}m\, f_0^{K^-p}(0) = (227 \pm 27)\,\mathrm{eV}. \qquad (4.22)$$

This agrees well with recent experimental data by the DEAR Collaboration $\Gamma_{1s}^{\rm exp} = (213 \pm 138)\,\mathrm{eV}$ [13].

4.2 Real part of $f_0^{K^-p}(0)_R$

A knowledge of the numerical values of the coupling constants g_1, g_2 and α_2 allows to calculate the real part of the amplitude $f_0^{K^-p}(0)_R$. In our approach it reads

$$\mathcal{R}e\, f_0^{K^-p}(0)_R = \frac{1}{2}\Big(\mathcal{R}e\, f_0^{K^-p}(0)_R^{I=0} + \mathcal{R}e\, f_0^{K^-p}(0)_R^{I=1}\Big) = \frac{1}{8\pi}\,\frac{\mu}{m_{K^-}}\Big[\frac{g_1^2}{m_{\Lambda_1^0} - m_{K^-} - m_p} + \frac{4}{9}\,\frac{g_2^2}{m_{\Sigma_2^0} - m_{K^-} - m_p}\Big] = (-0.154 \pm 0.009)\ \mathrm{fm}, \qquad (4.23)$$

where we have set $\alpha_2 = -1/3$.

Now we proceed to the analysis of the contribution of a smooth elastic background of low-energy elastic K^-p scattering.

5 Amplitude of low-energy K^-p scattering. Elastic background

At the hadronic level a smooth elastic background $A_B^{K^-p}$ we define as

$$A_B^{K^-p} = A_s^{K^-p} + A_t^{K^-p} + A_u^{K^-p}, \qquad (5.1)$$

where $A_s^{K^-p}$, $A_t^{K^-p}$ and $A_u^{K^-p}$ are the contributions of the s, t and u channels of low-energy elastic K^-p scattering, respectively.

For the calculation of the r.h.s. of (5.1) we assume the following contributions:

$$A_B^{K^-p} = A_{\rm CA}^{K^-p} + A_{\bar{K}K}^{K^-p}, \qquad (5.2)$$

where i) $A_{\rm CA}^{K^-p}$ is defined by the current algebra [36–38], accounting for all low-energy interactions which can be described by Effective Chiral Lagrangians [39]. In the general form this contribution has been calculated in [37,38]; ii) $A_{\bar{K}K}^{K^-p}$ is the contribution of the four-quark intermediate states $qq\bar{q}\bar{q}$ (or $\bar{K}K$ molecule) such as the scalar mesons $a_0(980)$, $f_0(980)$ and so on [40–44] (see also [45]) going beyond the scope of Effective Chiral Lagrangians. As has been recently found by the KLOE Collaboration (DAPHNE), measuring the radiative decays of the vector $\phi(1020)$-meson, $\phi(1020) \to a_0(980)\gamma$ and $\phi(1020) \to f_0(980)$, that the quark structure of the scalar mesons $a_0(980)$ and $f_0(980)$ differs substantially from $q\bar{q}$ [46].

5.1 Calculation of $A_{\rm CA}^{K^-p}$

The current algebra contribution to the parameter $A_B^{K^-p}$ we denote as

$$A_{\rm CA}^{K^-p} = \frac{1}{2}\,(A_0^0 + A_0^1), \qquad (5.3)$$

where A_0^0 and A_0^1 describe the contribution of K^-p scattering in the states with isospin $I = 0$ and $I = 1$. Using the results obtained in [37,38] we get

$$A_0^0 = \frac{3}{8\pi}\,\frac{\mu}{F_K^2}, \quad A_0^1 = \frac{1}{8\pi}\,\frac{\mu}{F_K^2}, \qquad (5.4)$$

where $F_K = 112.996\,\mathrm{MeV}$ is the PCAC constant of the $K^\pm$-meson [9]. This gives

$$A_{\rm CA}^{K^-p} = \frac{1}{4\pi}\,\frac{\mu}{F_K^2} = 0.398\,\mathrm{fm}. \qquad (5.5)$$

The value (5.5) is caused by the contributions of the s, t and u channels of low-energy elastic K^-p scattering, which can be described by Effective Chiral Lagrangians [39]. The result (5.5) is obtained at leading order in Chiral perturbation theory [22,23] (see also [47]). According to Chiral perturbation theory [22,23] the accuracy of the value, given by (5.5), is of order $O(m_{K^-}^2/16\pi^2F_K^2) = O(12\%)$. This coincides with an accuracy of the current algebra approach [48,49].

5.2 Calculation of the four-quark contribution $A_{\bar{K}K}^{K^-p}$

Four-quark states (or $\bar{K}K$ molecule) such as the scalar mesons $a_0(980)$ and $f_0(980)$ can give a contribution only to the t-channel of low-energy elastic K^-p scattering defined by the reaction $K^- + K^+ \to p + \bar{p}$. Since the four-quark states $a_0(980)$ and $f_0(980)$ cannot be described by Effective Chiral Lagrangians [39], the contribution of these states does not enter $A_{\rm CA}^{K^-p}$.

According to Jaffe [40], the scalar mesons $a_0(980)$ and $f_0(980)$ belong to an $SU(3)_{\rm flavour}$ nonet and the scalar meson $f_0(980)$ decouples from the $\pi\pi$ state. Following Jaffe [40], the $SU(3)_{\rm flavour}$ invariant interaction of the nonet of four-quark scalar mesons with two nonents of pseudoscalar light mesons, having a $q\bar{q}$ quark structure, can be written as

$$\mathcal{L}_{SPP}(x) = \sqrt{2}\,g_0\,{\rm tr}\{PPM\} = \sqrt{2}\,g_0\,P^b_a P^a_c M^c_b. \qquad (5.6)$$

where P and M are nonets of pseudoscalar light $q\bar{q}$ mesons and scalar $qq\bar{q}\bar{q}$ mesons, respectively,

$$P^b_a = \begin{pmatrix} \dfrac{\pi^0}{\sqrt{2}} + \dfrac{\eta_0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & -\dfrac{\pi^0}{\sqrt{2}} + \dfrac{\eta_0}{\sqrt{2}} & K^0 \\ -K^- & \bar{K}^0 & \eta_s \end{pmatrix},$$

$$M^b_a = \begin{pmatrix} \dfrac{a^0_0}{\sqrt{2}} - \dfrac{\varepsilon}{2} & a^+_0 & \kappa^+ \\ a^-_0 & -\dfrac{a^0_0}{\sqrt{2}} - \dfrac{\varepsilon}{2} & \kappa^0 \\ -\kappa^- & \bar{\kappa}^0 & -\dfrac{f_0}{\sqrt{2}} + \dfrac{\varepsilon}{2} \end{pmatrix}, \qquad (5.7)$$

where η_0 and η_s are pseudoscalar states with quark structure $\eta_0 = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $\eta_s = s\bar{s}$ [40]. Then, $\vec{a}_0 = (a^+_0, a^0_0, a^-_0) = (s\bar{s}u\bar{d}, s\bar{s}(u\bar{u} - d\bar{d})/\sqrt{2}, d\bar{u}s\bar{s})$ is the isotriplet of $a_0(980)$-mesons, $\kappa = (\kappa^+, \kappa^0) = (u\bar{s}d\bar{d}, d\bar{s}u\bar{u})$ and $\bar{\kappa} = (\bar{\kappa}^0, -\kappa^-) = (s\bar{d}u\bar{u}, -s\bar{u}d\bar{d})$ are doublets of strange scalar four-quark states, $f_0 = s\bar{s}(u\bar{u} - d\bar{d})/\sqrt{2}$ is the $f_0(980)$-meson and ε is the isoscalar scalar $\varepsilon(700)$-meson with $\varepsilon = u\bar{d}d\bar{u}$ quark structure and mass $m_\varepsilon = 700\,{\rm MeV}$ [40]. The nonet M is constructed in such a way that the $f_0(980)$-meson decouples from the $\pi\pi$ states, whereas the $\varepsilon(700)$-meson couples to the $\pi\pi$ states but decouples from the $\bar{K}K$ states. This implies that the $\varepsilon(700)$-meson does not contribute to the amplitude of K^-p scattering.

The interactions of the scalar mesons $a_0(980)$ and $f_0(980)$ with the K^--meson can be written as

$$\mathcal{L}_{SKK}(x) = g_0\,[-a^0_0(x) + f_0(x)]\,K^+(x)K^-(x), \qquad (5.8)$$

where $a^0_0(x)$, $f_0(x)$ and $K^\pm(x)$ are interpolating fields of the $a^0_0(980)$-, $f_0(980)$- and $K^\pm$-mesons.

For a numerical calculation we use the value $g_0 = g_{a_0K^+K^-} = g_{f_0K^+K^-} = 2.746\,{\rm GeV}$, obtained within the $\bar{K}K$ molecule model of the scalar mesons $a_0(980)$ and $f_0(980)$ [43] (see also [41] and [42]). In this model the scalar mesons $a_0(980)$ and $f_0(980)$ couple only to the $\bar{K}K$ states[8] and decouple from the $\pi\pi$ states.

The interaction of the nonet of four-quark scalar mesons M with octets of light baryons we define as

$$\mathcal{L}_{SBB}(x) = \sqrt{2}g_D\,{\rm tr}\{\{\bar{B}, B\}M\} + \sqrt{2}g_F\,{\rm tr}\{[\bar{B}, B]M\} = \sqrt{2}(g_D + g_F)\bar{B}^b_a B^a_c M^c_b + \sqrt{2}(g_D - g_F)\bar{B}^b_a B^c_b M^a_c, \qquad (5.9)$$

where B and $\bar{B}$ are octets of light baryons (see (4.2))

$$\bar{B}^b_a = \begin{pmatrix} \dfrac{\bar{\Sigma}^0}{\sqrt{2}} + \dfrac{\bar{\Lambda}^0}{\sqrt{6}} & \bar{\Sigma}^- & -\bar{\Xi}^- \\ \bar{\Sigma}^+ & -\dfrac{\bar{\Sigma}^0}{\sqrt{2}} + \dfrac{\bar{\Lambda}^0}{\sqrt{6}} & \bar{\Xi}^0 \\ \bar{p} & \bar{n} & -\dfrac{2}{\sqrt{6}}\bar{\Lambda}^0 \end{pmatrix} \qquad (5.10)$$

and g_D and g_F are the coupling constants of the symmetric and antisymmetric SBB interactions [50].

The effective Lagrangian of the SNN interaction reads

$$\begin{aligned}\mathcal{L}_{SBB}(x) = (g_D + g_F)\Big[&\sqrt{2}\,\bar{p}(x)n(x)a^+_0(x) \\ &+\sqrt{2}\,\bar{n}(x)p(x)a^-_0(x) + (\bar{p}(x)p(x) - \bar{n}(x)n(x))a^0_0(x) \\ &-(1 - 2\alpha_S)\,(\bar{p}(x)p(x) + \bar{n}(x)n(x))\,f_0(x) \\ &-\sqrt{2}\alpha_S\,(\bar{p}(x)p(x) + \bar{n}(x)n(x))\varepsilon(x)\Big] + \ldots, \qquad (5.11)\end{aligned}$$

where $\varepsilon(x)$, $p(x)$ and $n(x)$ are the interpolating fields of the $\varepsilon(700)$-meson, the proton and the neutron. The parameter α_S is given by $\alpha_S = g_F/(g_D + g_F)$ [50].

In order to suppress the contribution of the four-quark state $\varepsilon(700)$ to the S-wave scattering lengths of πN scattering we have to set $\alpha_S = 0$ or $g_F = 0$. As a result, the four-quark state $\varepsilon(700)$ decouples from nucleons. This gives

$$\begin{aligned}\mathcal{L}_{SBB}(x) = g_D\,\Big[&\sqrt{2}\,\bar{p}(x)n(x)a^+_0(x) + \sqrt{2}\,\bar{n}(x)p(x)a^-_0(x) \\ &+(\bar{p}(x)p(x) - \bar{n}(x)n(x))a^0_0(x) \\ &-(\bar{p}(x)p(x) + \bar{n}(x)n(x))f_0(x)\Big] + \ldots. \qquad (5.12)\end{aligned}$$

At the threshold of the reaction $K^- + p \to K^- + p$ the contribution of the four-quark states $a_0(980)$ and $f_0(980)$ we define as

$$A^{K^-p}_{\bar{K}K} = \frac{M(K^-p \to K^-p)_{a_0+f_0}}{8\pi(m_{K^-} + m_p)} = -\frac{g_D}{2\pi}\,\frac{g_0}{m^2_{a_0}}\,\frac{\mu}{m_{K^-}}, \qquad (5.13)$$

where we have set $m_{a_0} = m_{f_0} = 980\,{\rm MeV}$ [9].

The coupling constant g_D is not known [51]. For a further calculation of $A^{K^-p}_{\bar{K}K}$ we can set [52]

$$g_D = \frac{g_{\pi NN}}{g_A}\,\xi, \qquad (5.14)$$

where $g_{\pi NN} = 13.21$ [53] and $g_A = 1.267$ are the πNN coupling constant and the renormalization constant of the axial-vector coupling due to strong interactions, respectively, and ξ is a parameter, which we estimate below.

Using (5.14) the contribution of the $a_0(980)$ and $f_0(980)$ scalar mesons can be written as

$$A^{K^-p}_{\bar{K}K} = \frac{M(K^-p \to K^-p)_{a_0+f_0}}{8\pi(m_{K^-} + m_p)} = -\,\xi\,\frac{1}{2\pi}\,\frac{g_{\pi NN}}{g_A}\,\frac{g_0}{m^2_{a_0}}\,\frac{\mu}{m_{K^-}} = -0.614\,\xi\,{\rm fm}. \qquad (5.15)$$

[8] The scalar meson $a_0(980)$ couples also to the $\pi\eta$ states, where η is the well-known $\eta(550)$ pseudoscalar meson [9].

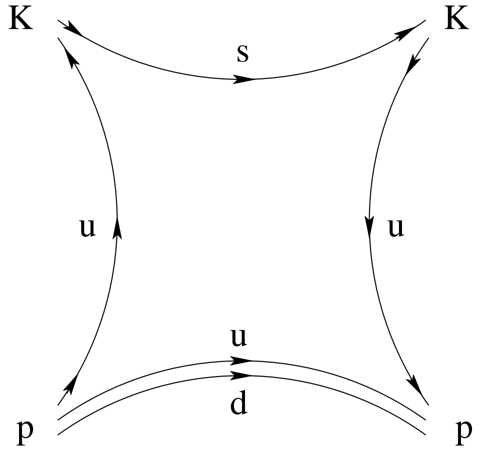

Fig. 1. The quark diagram describing a smooth elastic background of low-energy elastic K^-p scattering in the Effective quark model with chiral $U(3)\times U(3)$ symmetry.

The parameter $A_B^{K^-p}$ is equal to

$$A_B^{K^-p} = 0.398 - 0.614\,\xi\,\mathrm{fm}. \tag{5.16}$$

In order estimate the value of the parameter ξ we suggest to calculate the parameter $A_B^{K^-p}$ within the Effective quark model with chiral $U(3)\times U(3)$ symmetry [17–19].

5.3 $A_B^{K^-p}$ in the Effective quark model with chiral $U(3)\times U(3)$ symmetry

Following the principle of the quark-hadron duality [54] we assume that the contribution of the smooth elastic background of low-energy elastic K^-p scattering can be fully fitted by the lowest quark box-diagram depicted in fig. 1, calculated with the Effective quark model with chiral $U(3)\times U(3)$ symmetry [17–19].

Using the reduction technique [21] the amplitude of elastic low-energy K^-p scattering we define as

$$\begin{aligned}&(2\pi)^4 i\,\delta^{(4)}(q\,' + p\,' - q - p)\,M(K^-p\to K^-p) = \\ &\lim_{p\,'^2,\,p^2\to m_p^2,\,q\,'^2,\,q^2\to m^2_{K^-}}\int \mathrm{d}^4x_1\mathrm{d}^4x_2\mathrm{d}^4x_3\mathrm{d}^4x_4 \\ &\quad\times e^{i\,q\,'\cdot x_1 + ip\,'\cdot x_2 - ip\cdot x_3 - iq\cdot x_4} \\ &\quad\times(\Box_1 + m^2_{K^-})(\Box_4 + m^2_{K^-})\,\bar u(p\,',\sigma\,')\,\overrightarrow{(i\gamma_\nu\,\partial_2^\nu - m_p)} \\ &\quad\times\langle 0|\mathrm{T}(K^-(x_1)p(x_2)\bar p(x_3)K^+(x_4))|0\rangle \\ &\quad\times\overleftarrow{(-i\gamma_\mu\,\partial_3^\mu - m_p)}\,u(p,\sigma),\end{aligned} \tag{5.17}$$

where $p(x)$ and $u(p,\sigma)$ are the interpolating field operator and the Dirac bispinor of the proton, respectively, and $K^\pm(x)$ are the interpolating fields of the $K^\mp$-mesons.

In order to describe the r.h.s. of eq. (5.17) at the quark level we follow [17] and use the equations of motion

$$\begin{aligned}\overrightarrow{(i\gamma_\nu\,\partial_2^\nu - m_p)}\,p(x_1) &= \frac{g_B}{\sqrt 2}\,\eta_p(x_2),\\ \bar p(x_3)\overleftarrow{(-i\gamma_\mu\partial_3^\mu - m_p)} &= \frac{g_B}{\sqrt 2}\,\bar\eta_p(x_3),\end{aligned} \tag{5.18}$$

where $\eta_p(x_2)$ and $\bar\eta_p(x_3)$ are the three-quark current densities [17]

$$\begin{aligned}\eta_p(x_2) &= -\,\varepsilon^{ijk}\,[\bar{u^c}_i(x_2)\gamma^\mu u_j(x_2)]\gamma_\mu\gamma^5 d_k(x_2),\\ \bar\eta_p(x_3) &= +\,\varepsilon^{ijk}\,\bar d_i(x_3)\gamma^\mu\gamma^5[\bar u_j(x_3)\gamma_\mu u^c_k(x_3)]\,,\end{aligned} \tag{5.19}$$

where i,j and k are colour indices and $\bar\psi^{\,c}(x) = \psi(x)^T C$ and $C = -C^T = -C^\dagger = -C^{-1}$ is the charge conjugate matrix, T denotes transposition, and g_B is the phenomenological coupling constant of the low-lying baryon octet $B_8(x)$ coupled to the three-quark current densities [17]

$$\mathcal{L}^{(B)}_{\mathrm{int}}(x) = \frac{g_B}{\sqrt 2}\,\bar B_8(x)\eta_8(x) + \mathrm{h.c.} \tag{5.20}$$

The coupling constant g_B is equal to $g_B = 1.34\times 10^{-4}\,\mathrm{MeV}^{-2}$ [17].

For the interpolating field operators of the $K^\pm$-mesons we use the following equations of motion [17]:

$$\begin{aligned}(\Box_1 + m^2_{K^-})K^-(x_1) &= \frac{g_K}{\sqrt 2}\,\bar u(x_1)i\gamma^5 s(x_1),\\ (\Box_4 + m^2_{K^-})K^+(x_4) &= \frac{g_K}{\sqrt 2}\,\bar s(x_4)i\gamma^5 u(x_4),\end{aligned} \tag{5.21}$$

where $g_K = (m+m_s)/\sqrt 2 F_K$, $m = 330\,\mathrm{MeV}$ and $m_s = 465\,\mathrm{MeV}$ are the masses of the constituent u, d and s quarks, respectively [17,19] (see also [55]).

The amplitude of low-energy elastic K^-p scattering is defined by

$$\begin{aligned}&M(K^-p\to K^-p) = \\ &-i\,\frac14\,g_B^2\,g_K^2\int \mathrm{d}^4x_1\mathrm{d}^4x_2\mathrm{d}^4x_3\,e^{i\,q\,'\cdot x_1 + ip\,'\cdot x_2 - ip\cdot x_3}\,\bar u(p\,',\sigma\,') \\ &\times\langle 0|\mathrm{T}(\bar u(x_1)i\gamma^5 s(x_1)\eta_p(x_2)\bar\eta_p(x_3)\bar s(0)i\gamma^5 u(0))|0\rangle u(p,\sigma),\end{aligned} \tag{5.22}$$

where the external momenta $q\,'$, $p\,'$, q and p should be kept on mass shell $q\,'^2 = q^2 = m^2_{K^-}$ and $p\,'^2 = p^2 = m_p^2$.

In the appendix we have carried out the calculation of the amplitude (5.22) at threshold. The parameter $A_B^{K^-p}$ is equal to (see (A.9))

$$A_B^{K^-p} = \frac{M(K^-p\to K^-p)}{8\pi(m_{K^-} + m_p)} = -0.328 \pm 0.033\,\mathrm{fm}. \tag{5.23}$$

This allows to estimate the value of the parameter ξ (5.14). Equating (5.16) to (5.23) we get $\xi = 1.2\pm 0.1$.

5.4 S-wave scattering length $a_0^{K^-p}$ and shift ϵ_{1s}^{th}

Using the value of the parameter $A_B^{K^-p}$, describing the contribution of the smooth elastic background of low-energy elastic K^-p scattering, we obtain the S-wave scattering length $a_0^{K^-p}$:

$$a_0^{K^-p} = (-0.328 \pm 0.033) + (-0.154 \pm 0.009) = (-0.482 \pm 0.034)\,\mathrm{fm}. \quad (5.24)$$

This results in the shift of the energy level of the ground state of kaonic hydrogen

$$\epsilon_{1s}^{\rm th} = -\,421.124\, a_0^{K^-p} = 203 \pm 15\ \mathrm{eV}. \quad (5.25)$$

The theoretical value fits well the preliminary experimental data $\epsilon_{1s}^{\rm exp} = (183 \pm 62)\,\mathrm{eV}$ by the DEAR Collaboration [13].

6 Electromagnetic decay channels

It is well known [53] that in the case of the energy level displacement of the ground state of pionic hydrogen the electromagnetic channel $A_{\pi p} \to n + \gamma$ defines 64% of the experimental value of the width $\Gamma_{1s} = (0.868 \pm 0.056)\,\mathrm{eV}$. The width of the energy level of the ground state of pionic hydrogen can be written as

$$\Gamma_{1s} = \frac{8\pi}{9}\,\frac{p^*}{\mu}\,\Big(a_0^{1/2} - a_0^{3/2}\Big)^2\,|\Psi_{1s}(0)|^2\Big(1 + \frac{1}{P}\Big), \quad (6.1)$$

where $\mu = m_{\pi^-}m_p/(m_{\pi^-} + m_p) = 121.497\,\mathrm{MeV}$ is the reduced mass of the π^-p system for $m_{\pi^-} = 139.570\,\mathrm{MeV}$ and $m_p = 938.272\,\mathrm{MeV}$, p^* is the relative momentum equal to

$$p^* = \frac{m_p + m_{\pi^-}}{2} \times\sqrt{\Big[1 - \Big(\frac{m_n + m_{\pi^0}}{m_p + m_{\pi^-}}\Big)^2\Big]\Big[1 - \Big(\frac{m_n - m_{\pi^0}}{m_p + m_{\pi^-}}\Big)^2\Big]} = 28.040\,\mathrm{MeV}, \quad (6.2)$$

$\Psi_{1s}(0) = 1/\sqrt{\pi a_{\rm B}^3}$ is the wave function of the ground state of pionic hydrogen at the origin, and $a_0^{1/2}$ and $a_0^{3/2}$ are the S-wave scattering lengths of πN scattering with isospin $I = 1/2$ and $I = 3/2$. The experimental values $a_0^{1/2} = 0.1788 \pm 0.0043\, m_{\pi^-}^{-1}$ and $a_0^{3/2} = -0.0927 \pm 0.0085\, m_{\pi^-}^{-1}$, obtained by the PSI Collaboration [53], give $a_0^{1/2} - a_0^{3/2} = 0.2715 \pm 0.0095\, m_{\pi^-}^{-1}$. Then, P is the Panofsky ratio defined by [16]

$$\frac{1}{P} = \frac{\Gamma(A_{\pi p} \to n\gamma)}{\Gamma(A_{\pi p} \to n\pi^0)} = 0.647 \pm 0.004, \quad (6.3)$$

where we have adduced the experimental value of $1/P$ obtained in [16].

In the case of kaonic hydrogen there are two electromagnetic decay channels $A_{Kp} \to \Lambda^0 + \gamma$ and $A_{Kp} \to \Sigma^0 + \gamma$, which are related to the reactions $K^- + p \to \Lambda^0 + \gamma$ and $K^- + p \to \Sigma^0 + \gamma$. Therefore, the total width of the energy level of the ground state of kaonic hydrogen can be written as [15]

$$\Gamma_{1s} = \frac{4\pi}{\mu}\,\mathcal{I}m\, f_{K^-p}(0)\,|\Psi_{1s}(0)|^2\,(1 + X), \quad (6.4)$$

where X, the inverse Panofsky ratio for kaonic hydrogen, is defined by [15]

$$X = \frac{\Gamma(A_{Kp} \to \Lambda^0\gamma) + \Gamma(A_{Kp} \to \Sigma^0\gamma)}{\Gamma_{1s}}. \quad (6.5)$$

Below we give a theoretical analysis and numerical estimate of the value of X.

First, we consider the decay of pionic hydrogen $A_{\pi p} \to n + \gamma$, then we extend the developed technique and methodics to the decays of kaonic hydrogen $A_{Kp} \to \Lambda^0 + \gamma$ and $A_{Kp} \to \Sigma^0 + \gamma$.

6.1 Radiative decay of pionic hydrogen

The amplitude of the decay $A_{\pi p} \to n + \gamma$ we define as [7, 8, 48, 49]

$$M(A_{\pi p} \to n\gamma) = \sqrt{\frac{1}{2\mu}}\int\frac{d^3k}{(2\pi)^3}\sqrt{\frac{m_{\pi^-}m_p}{E_{\pi^-}(\vec{k}\,)E_p(\vec{k}\,)}} \times \Phi_{1s}(\vec{k}\,)\,M(\pi^-(\vec{k}\,)p(-\vec{k}\,) \to n\gamma), \quad (6.6)$$

where $\mu = m_{\pi^-}m_p/(m_{\pi^-} + m_p) = 121.497\,\mathrm{MeV}$ is the reduced mass of the π^-p system and $\Phi_{1s}(\vec{k}\,)$ is the wave function of the ground state of pionic hydrogen in the momentum representation.

The amplitude $M(\pi^-(\vec{k}\,)p(-\vec{k}\,) \to n\gamma)$ of the reaction $\pi^- + p \to n + \gamma$ is determined by [48, 49]

$$M(\pi^-(\vec{k}\,)p(-\vec{k}\,) \to n\gamma) = \sqrt{4\pi}\,e\,\langle n(-\vec{q},\sigma)|J_\mu^{\rm e\ell m}(0)|\pi^-(\vec{k}\,)p(-\vec{k},\sigma_p)\rangle\, e^\mu(\vec{q},\lambda), \quad (6.7)$$

where $J_\mu^{\rm e\ell m}(0)$ is the electromagnetic hadronic current [48, 49]

$$J_\mu^{\rm e\ell m}(0) = J_\mu^3(0) + \frac{1}{\sqrt{3}}\,J_\mu^8(0). \quad (6.8)$$

Here, $J_\mu^3(0)$ is the third component of the isotopic vector and $J_\mu^8(0)$, the isospin singlet, is the eighth component of the $SU(3)_{\rm flavour}$ octet; $e^\mu(\vec{q},\lambda)$ is the polarization vector of the emitted photon.

Using the reduction technique [21] for the π^--meson we reduce the matrix element of the electromagnetic hadronic current (6.6) to the form

$$\langle n(-\vec{q},\sigma)|J^{\mathrm{e}\ell\mathrm{m}}_{\mu}(0)|\pi^-(\vec{k}\,)p(-\vec{k},\sigma_p)\rangle = \lim_{k^2_{\pi^-}\to m^2_{\pi^-}} i\int \mathrm{d}^4x\, e^{-ik_{\pi^-}\cdot x}\,(\Box_x + m^2_{\pi^-}) \times\langle n(-\vec{q},\sigma)|\mathrm{T}(J^{\mathrm{e}\ell\mathrm{m}}_{\mu}(0)\pi^{-\dagger}(x))|p(-\vec{k},\sigma_p)\rangle, \quad (6.9)$$

where $k_{\pi^-} = \left(\sqrt{\vec{k}^{\,2} + m^2_{\pi^-}}, \vec{k}\right)$. According to the PCAC hypothesis [48,49] the interpolating fields of the π-mesons are related to the divergences of the axial-vector currents. For the π^--meson field we get

$$\pi^{-\dagger}(x) = \frac{1}{\sqrt{2}}\,\frac{1}{m^2_{\pi}F_{\pi}}\,\partial^{\nu}J^{1-i2}_{5\nu}(x), \quad (6.10)$$

where $F_{\pi} = 92.419\,\mathrm{MeV}$ is the PCAC constant and $J^{1-i2}_{5\nu}(x) = J^1_{5\nu}(x) - iJ^2_{5\nu}(x)$ is the hadronic axial-vector current [48,49].

In the soft-pion limit [48,49] the r.h.s. of (6.9) can be rewritten as[9]

$$\langle n(-\vec{q},\sigma)|J^{\mathrm{e}\ell\mathrm{m}}_{\mu}(0)|\pi^-(\vec{k}\,)p(-\vec{k},\sigma_p)\rangle = \frac{i}{\sqrt{2}F_{\pi}}\int \mathrm{d}^4x\,\langle n(-\vec{q},\sigma)|\mathrm{T}(J^{\mathrm{e}\ell\mathrm{m}}_{\mu}(0)\partial^{\nu}J^{1-i2}_{5\nu}(x))|p(\vec{0},\sigma_p)\rangle. \quad (6.11)$$

Integrating by parts we arrive at the expression [48,49]

$$\langle n(-\vec{q},\sigma)|J^{\mathrm{e}\ell\mathrm{m}}_{\mu}(0)|\pi^-(\vec{k}\,)p(-\vec{k},\sigma_p)\rangle = \frac{i}{\sqrt{2}F_{\pi}}\langle n(-\vec{q},\sigma)|[J^{\mathrm{e}\ell\mathrm{m}}_{\mu}(0), Q^{1-i2}_5(0)]|p(\vec{0},\sigma_p)\rangle, \quad (6.12)$$

where $Q^{1-i2}_{5\nu}(0)$ is the axial-vector charge operator

$$Q^{1-i2}_5(0) = \int \mathrm{d}^3x\, J^{1-i2}_{50}(0,\vec{x}\,). \quad (6.13)$$

Using Gell-Mann's current algebra [48,49] we get

$$\langle n(-\vec{q},\sigma)|J^{\mathrm{e}\ell\mathrm{m}}_{\mu}(0)|\pi^-(\vec{k}\,)p(-\vec{k},\sigma_p)\rangle = -\frac{i}{\sqrt{2}F_{\pi}}\langle n(-\vec{q},\sigma)|J^{1-i2}_{5\mu}(0)|p(\vec{0},\sigma_p)\rangle. \quad (6.14)$$

The matrix element in the r.h.s. of (6.14) is related to the matrix element of the axial-vector current defining the β-decay of the neutron [56,57]

$$\langle n(-\vec{q},\sigma)|J^{1-i2}_{5\mu}(0)|p(\vec{0},\sigma_p)\rangle = g_A\,\bar{u}_n(-\vec{q},\sigma)\gamma_{\mu}\gamma^5 u(\vec{0},\sigma_p), \quad (6.15)$$

where $\bar{u}_n(-\vec{q},\sigma)$ and $u(\vec{0},\sigma_p)$ are Dirac bispinors of the neutron and the proton, respectively.

[9] The soft-pion limit as well as the soft-kaon limit should be understood as ChPT at leading order in chiral expansions [22,23].

Thus, the matrix element of the reaction $\pi^- + p \to n + \gamma$ is determined by

$$M(\pi^-(\vec{k}\,)p(-\vec{k}\,) \to n\gamma) = -\sqrt{2\pi}\,\frac{ieg_A}{F_{\pi}}\,\bar{u}(-\vec{q},\sigma)\gamma_{\mu}\gamma^5 u(\vec{0},\sigma_p)\,e^{\mu}(\vec{q},\lambda). \quad (6.16)$$

The partial width of the decay $A_{\pi p} \to n + \gamma$ is equal to

$$\Gamma(A_{\pi p} \to n\gamma) = \alpha\,\frac{3}{4}\,\frac{g^2_A}{F^2_{\pi}}\,\frac{m_n}{m_{\pi^-}} \times\left(1 - \frac{m^2_n}{(m_{\pi^-} + m_p)^2}\right)|\Psi_{1s}(0)|^2 = 0.369\,\mathrm{eV}. \quad (6.17)$$

This value should be compared with the partial width of the decay $A_{\pi p} \to n\pi^0$, which reads

$$\Gamma(A_{\pi p} \to n\pi^0) = \frac{8\pi}{9}\,\frac{p^*}{\mu} \times\left(a^{1/2}_0 - a^{3/2}_0\right)^2|\Psi_{1s}(0)|^2 = 0.542\,\mathrm{eV}. \quad (6.18)$$

The Panofsky ratio $1/P$ is equal to

$$\frac{1}{P} = \frac{27}{32}\,\frac{\alpha}{\pi}\,\frac{g^2_A}{F^2_{\pi}}\,\frac{m_n}{m_{\pi^-}}\,\frac{\mu}{p^*}\,\frac{1}{\left(a^{1/2}_0 - a^{3/2}_0\right)^2} \times\left(1 - \frac{m^2_n}{(m_{\pi^-} + m_p)^2}\right) = 0.681 \pm 0.048. \quad (6.19)$$

The theoretical value agrees with the experimental data $1/P = 0.647 \pm 0.004$ [16]. The theoretical error is related to the errors of the experimental values of the S-wave scattering lengths $a^{1/2}_0 - a^{3/2}_0 = (0.2715 \pm 0.0095)\,m^{-1}_{\pi^-}$ [53].

The cross-section for the reaction $\pi^- + p \to n + \gamma$ at low relative velocities of the π^-p system v is equal to

$$\sigma(\pi^-p \to n\gamma) = \frac{432}{v}\;\mu\mathrm{barn}. \quad (6.20)$$

The result (6.20) agrees well with the theoretical estimate given by Anderson and Fermi [58].

Now we are able to apply the technique developed above to the calculation of the partial widths of the electromagnetic decay channels of kaonic hydrogen $A_{Kp} \to \Lambda^0 + \gamma$ and $A_{Kp} \to \Sigma^0 + \gamma$.

6.2 Radiative decays of kaonic hydrogen

Amplitudes of the decays $A_{Kp} \to \Lambda^0 + \gamma$ and $A_{Kp} \to \Sigma^0 + \gamma$ we define in analogy to (6.6). This gives

$$\begin{aligned} M(A_{Kp} \to \Lambda^0\gamma) &= \sqrt{\frac{1}{2\mu}} \int \frac{d^3k}{(2\pi)^3} \\ &\times \sqrt{\frac{m_{K^-} m_p}{E_{K^-}(\vec{k}\,)E_p(\vec{k}\,)}}\, \Phi_{1s}(\vec{k}\,)\, M(K^-(\vec{k}\,)p(-\vec{k}\,) \to \Lambda^0\gamma), \\ M(A_{Kp} \to \Sigma^0\gamma) &= \sqrt{\frac{1}{2\mu}} \int \frac{d^3k}{(2\pi)^3} \\ &\times \sqrt{\frac{m_{K^-} m_p}{E_{K^-}(\vec{k}\,)E_p(\vec{k}\,)}}\, \Phi_{1s}(\vec{k}\,)\, M(K^-(\vec{k}\,)p(-\vec{k}\,) \to \Sigma^0\gamma), \end{aligned} \tag{6.21}$$

where $\mu = m_{K^-} m_p/(m_{K^-} + m_p) = 323.478\,\mathrm{MeV}$ is the reduced mass of the K^-p system and $\Phi_{1s}(\vec{k}\,)$ is the wave function of the ground state of kaonic hydrogen in the momentum representation.

The amplitudes of the reactions $K^- + p \to \Lambda^0 + \gamma$ and $K^- + p \to \Sigma^0 + \gamma$ read

$$\begin{aligned} &M(K^-(\vec{k}\,)p(-\vec{k}\,) \to \Lambda^0\gamma) = \\ &\sqrt{4\pi}\, e\, \langle \Lambda^0(-\vec{q},\sigma)|J^{\mathrm{e}\ell\mathrm{m}}_\mu(0)|K^-(\vec{k}\,)p(-\vec{k},\sigma_p)\rangle\, e^\mu(\vec{q},\lambda), \\ &M(K^-(\vec{k}\,)p(-\vec{k}\,) \to \Sigma^0\gamma) = \\ &\sqrt{4\pi}\, e\, \langle \Sigma^0(-\vec{q},\sigma)|J^{\mathrm{e}\ell\mathrm{m}}_\mu(0)|K^-(\vec{k}\,)p(-\vec{k},\sigma_p)\rangle\, e^\mu(\vec{q},\lambda). \end{aligned} \tag{6.22}$$

The application of the reduction technique reduces the matrix elements (6.22) to the form

$$\begin{aligned} &\langle \Lambda^0(-\vec{q},\sigma)|J^{\mathrm{e}\ell\mathrm{m}}_\mu(0)|K^-(\vec{k}\,)p(-\vec{k},\sigma_p)\rangle = \\ &\qquad \lim_{k^2_{K^-} \to m^2_{K^-}} i \int d^4x\, e^{-ik_{K^-}\cdot x}\, (\Box_x + m^2_{K^-}) \\ &\qquad \times \langle \Lambda^0(-\vec{q},\sigma)|\mathrm{T}(J^{\mathrm{e}\ell\mathrm{m}}_\mu(0)K^{-\dagger}(x))|p(-\vec{k},\sigma_p)\rangle, \\ &\langle \Sigma^0(-\vec{q},\sigma)|J^{\mathrm{e}\ell\mathrm{m}}_\mu(0)|K^-(\vec{k}\,)p(-\vec{k},\sigma_p)\rangle = \\ &\qquad \lim_{k^2_{K^-} \to m^2_{K^-}} i \int d^4x\, e^{-ik_{K^-}\cdot x}\, (\Box_x + m^2_{K^-}) \\ &\qquad \times \langle \Sigma^0(-\vec{q},\sigma)|\mathrm{T}(J^{\mathrm{e}\ell\mathrm{m}}_\mu(0)K^{-\dagger}(x))|p(-\vec{k},\sigma_p)\rangle. \end{aligned} \tag{6.23}$$

The PCAC hypothesis allows to define the interpolating field $K^{-\dagger}(x)$ in terms of the divergence of the axial-vector current [48,49]

$$K^{-\dagger}(x) = \frac{1}{\sqrt{2}}\, \frac{1}{m^2_{K^-} F_K}\, \partial^\nu J^{4-i5}_{5\nu}(x). \tag{6.24}$$

In the soft-kaon limit $k_{K^-} \to 0$ we obtain

$$\begin{aligned} &\langle \Lambda^0(-\vec{q},\sigma)|J^{\mathrm{e}\ell\mathrm{m}}_\mu(0)|K^-(\vec{k}\,)p(-\vec{k},\sigma_p)\rangle = \\ &\quad \frac{i}{\sqrt{2}F_K}\langle \Lambda^0(-\vec{q},\sigma)|[J^{\mathrm{e}\ell\mathrm{m}}_\mu(0), Q^{4-i5}_5(0)]|p(\vec{0},\sigma_p)\rangle, \\ &\langle \Sigma^0(-\vec{q},\sigma)|J^{\mathrm{e}\ell\mathrm{m}}_\mu(0)|K^-(\vec{k}\,)p(-\vec{k},\sigma_p)\rangle = \\ &\quad \frac{i}{\sqrt{2}F_K}\langle \Sigma^0(-\vec{q},\sigma)|[J^{\mathrm{e}\ell\mathrm{m}}_\mu(0), Q^{4-i5}_5(0)]|p(\vec{0},\sigma_p)\rangle. \end{aligned} \tag{6.25}$$

Using Gell-Mann's current algebra [48,49] we transcribe the r.h.s. of the matrix elements (6.25) into the form

$$\begin{aligned} &\langle \Lambda^0(-\vec{q},\sigma)|J^{\mathrm{e}\ell\mathrm{m}}_\mu(0)|K^-(\vec{k}\,)p(-\vec{k},\sigma_p)\rangle = \\ &\quad -\frac{i}{\sqrt{2}F_K}\langle \Lambda^0(-\vec{q},\sigma)|J^{4-i5}_{5\mu}(0)|p(\vec{0},\sigma_p)\rangle, \\ &\langle \Sigma^0(-\vec{q},\sigma)|J^{\mathrm{e}\ell\mathrm{m}}_\mu(0)|K^-(\vec{k}\,)p(-\vec{k},\sigma_p)\rangle = \\ &\quad -\frac{i}{\sqrt{2}F_K}\langle \Sigma^0(-\vec{q},\sigma)|J^{4-i5}_{5\mu}(0)|p(\vec{0},\sigma_p)\rangle, \end{aligned} \tag{6.26}$$

where $F_K = 112.996\,\mathrm{MeV}$ is the PCAC constant of $K^\pm$-mesons [9]. The matrix elements of the axial-vector current in the r.h.s. of (6.26) can be defined in analogy with (6.15)

$$\begin{aligned} &\langle \Lambda^0(-\vec{q},\sigma)|J^{4-i5}_{5\mu}(0)|p(\vec{0},\sigma_p)\rangle = \\ &\quad g^{\Lambda^0}_A\, \bar{u}_{\Lambda^0}(-\vec{q},\sigma)\gamma_\mu\gamma^5 u(\vec{0},\sigma_p), \\ &\langle \Sigma^0(-\vec{q},\sigma)|J^{4-i5}_{5\mu}(0)|p(\vec{0},\sigma_p)\rangle = \\ &\quad g^{\Sigma^0}_A\, \bar{u}_{\Sigma^0}(-\vec{q},\sigma)\gamma_\mu\gamma^5 u(\vec{0},\sigma_p). \end{aligned} \tag{6.27}$$

The partial widths of the decays $A_{Kp} \to \Lambda^0\gamma$ and $A_{Kp} \to \Sigma^0\gamma$ are equal to

$$\begin{aligned} \Gamma(A_{Kp} \to \Lambda^0\gamma) &= \alpha\, \frac{3}{4}\, \frac{(g^{\Lambda^0}_A)^2}{F^2_K}\, \frac{m_{\Lambda^0}}{m_{K^-}} \\ &\times \left(1 - \frac{m^2_{\Lambda^0}}{(m_{K^-} + m_p)^2}\right) |\Psi_{1s}(0)|^2, \\ \Gamma(A_{Kp} \to \Sigma^0\gamma) &= \alpha\, \frac{3}{4}\, \frac{(g^{\Sigma^0}_A)^2}{F^2_K}\, \frac{m_{\Sigma^0}}{m_{K^-}} \\ &\times \left(1 - \frac{m^2_{\Sigma^0}}{(m_{K^-} + m_p)^2}\right) |\Psi_{1s}(0)|^2. \end{aligned} \tag{6.28}$$

The coupling constant $g^{\Lambda^0}_A$ can be taken from the data on the β-decay of the Λ^0-hyperon, $\Lambda^0 \to p + e^- + \bar{\nu}_e$: $g^{\Lambda^0}_A = 0.718 \pm 0.015$ [9]. Due to isospin invariance of strong interactions we can set $g^{\Sigma^0}_A = g^{\Sigma^-}_A/\sqrt{2} = 0.240 \pm 0.012$ [59], where $g^{\Sigma^-}_A = 0.340 \pm 0.017$ defines the β-decay $\Sigma^- \to n + e^- + \bar{\nu}_e$ [9]. As a result, we obtain the following numerical values of the partial widths:

$$\begin{aligned} \Gamma(A_{Kp} \to \Lambda^0\gamma) &= (0.82 \pm 0.04)\,\mathrm{eV}, \\ \Gamma(A_{Kp} \to \Sigma^0\gamma) &= (0.08 \pm 0.01)\,\mathrm{eV}, \end{aligned} \tag{6.29}$$

where we have used $m_{\Lambda^0} = 1115.683\,\mathrm{MeV}$ and $m_{\Sigma^0} = 1192.642\,\mathrm{MeV}$ [9].

The parameter X, the inverse Panofsky ratio for kaonic hydrogen, is equal to

$$X = \frac{\Gamma(A_{Kp} \to \Lambda^0\gamma) + \Gamma(A_{Kp} \to \Sigma^0\gamma)}{\Gamma_{1s}} = \alpha\,\frac{3}{16\pi}\,\frac{1}{F_K^2}\,\frac{\mu}{m_{K^-}}\,\frac{1}{\mathcal{I}m\,f_0^{K^-p}(0)} \times\Bigg[(g_A^{\Lambda^0})^2\, m_{\Lambda^0}\,\Big(1 - \frac{m^2_{\Lambda^0}}{(m_{K^-} + m_p)^2}\Big) +(g_A^{\Sigma^0})^2 m_{\Sigma^0}\,\Big(1 - \frac{m^2_{\Sigma^0}}{(m_{K^-} + m_p)^2}\Big)\Bigg] = (3.97 \pm 0.47) \times 10^{-3}. \qquad (6.30)$$

Thus, the contribution of radiative decay channels $A_{Kp} \to \Lambda^0\gamma$ and $A_{Kp} \to \Sigma^0\gamma$ to the width of the ground state of kaonic hydrogen is less than 0.5%.

The branching ratios $\mathrm{B}(A_{Kp} \to \Lambda^0\gamma) = (3.61\pm0.43)\times 10^{-3}$ and $\mathrm{B}(A_{Kp} \to \Sigma^0\gamma) = (0.35 \pm 0.04) \times 10^{-3}$, obtained for the partial widths (6.29) and the total width $\Gamma_{1s} = (227 \pm 27)\,\mathrm{eV}$ given by (4.22), are in qualitative agreement with both theoretical values, predicted by Hamaie *et al.* [60], $\mathrm{B}(A_{Kp} \to \Lambda^0\gamma) = 4.72 \times 10^{-3}$ and $\mathrm{B}(A_{Kp} \to \Sigma^0\gamma) = 2.43 \times 10^{-3}$, and experimental values, $\mathrm{B}(A_{Kp} \to \Lambda^0\gamma) = (0.86 \pm 0.12) \times 10^{-3}$ and $\mathrm{B}(A_{Kp} \to \Sigma^0\gamma) = (1.44 \pm 0.23) \times 10^{-3}$ [61].

The branching ratio of the radiative decays of the $\Lambda(1405)$-resonance is equal to $\mathrm{B}(\Lambda(1405) \to \Lambda^0\gamma) + \mathrm{B}(\Lambda(1405) \to \Sigma^0\gamma) = (0.13 \pm 0.03)\%$ [9,62]. The data on radiative decays of the $\Sigma(1750)$-resonance are absent [9].

Hence, within an accuracy about 1% one can neglect the contributions of radiative decay channels to the width of the ground state of kaonic hydrogen.

7 Conclusion

We have analysed the energy level displacement of the ground state of kaonic hydrogen within a quantum field theoretic and relativistic covariant approach. In our approach the energy level displacement of the ground state of kaonic hydrogen is defined by the amplitude of the reaction $K^- + p \to K^- + p$, weighted with the wave functions of kaonic hydrogen in the ground state (3.5). It reads

$$-\epsilon_{1s} + i\,\frac{\Gamma_{1s}}{2} = \frac{1}{4m_{K^-}m_p}\int\frac{\mathrm{d}^3k}{(2\pi)^3}\int\frac{\mathrm{d}^3q}{(2\pi)^3} \times\sqrt{\frac{m_{K^-}m_p}{E_{K^-}(\vec{k}\,)E_p(\vec{k}\,)}}\sqrt{\frac{m_{K^-}m_p}{E_{K^-}(\vec{q}\,)E_p(\vec{q}\,)}} \times\Phi^{\dagger}_{1s}(\vec{k}\,)\,M(K^-(\vec{q}\,)p(-\vec{q},\sigma_p) \to K^-(\vec{k}\,)p(-\vec{k},\sigma_p))\,\Phi_{1s}(\vec{q}\,). \qquad (7.1)$$

By virtue of the wave functions $\Phi^{\dagger}_{1s}(\vec{k}\,)$ and $\Phi_{1s}(\vec{q}\,)$ the integrand is concentrated around momenta $k \sim 1/a_{\mathrm{B}}$ and $q \sim 1/a_{\mathrm{B}}$, where $1/a_{\mathrm{B}} = 2.361\,\mathrm{MeV}$. Since typical momenta are much less than the masses of coupled particles, $m_{K^-} \gg 1/a_{\mathrm{B}}$ and $m_p \gg 1/a_{\mathrm{B}}$, the zero-momentum limit $k = q = 0$ turns out to be a good approximation[10]. This results in the well-known DGBT formula

$$-\epsilon_{1s} + i\,\frac{\Gamma_{1s}}{2} = 2\alpha^3\mu^2\,f_0^{K^-p}(0), \qquad (7.2)$$

where $f_0^{K^-p}(0)$ is the partial S-wave amplitude of the reaction $K^- + p \to K^- + p$ at threshold.

For the description of the amplitude $f_0^{K^-p}(0)$ we have suggested the dominance of a smooth elastic background of low-energy K^-p scattering and three resonances $\Lambda(1405)$, the $SU(3)_{\mathrm{flavour}}$ singlet, and the $\Lambda(1800)$ and $\Sigma(1750)$, the components of the $SU(3)_{\mathrm{flavour}}$ octet. These resonances saturate the part of the amplitude which we have denoted as $f_0^{K^-p}(0)_R$ (3.10).

The imaginary part of the amplitude $f_0^{K^-p}(0)_R$ is related to inelastic channels $K^-p \to \Sigma^-\pi^+$, $K^-p \to \Sigma^+\pi^-$, $K^-p \to \Sigma^0\pi^0$ and $K^-p \to \Lambda^0\pi^0$, which are fully described by the resonances $\Lambda(1405)$, $\Lambda(1800)$ and $\Sigma(1750)$.

For the analysis of the consistency of our approach, applied to the description of inelastic channels $K^-p \to \Sigma^-\pi^+$, $K^-p \to \Sigma^+\pi^-$, $K^-p \to \Sigma^0\pi^0$ and $K^-p \to \Lambda^0\pi^0$, we have used the experimental data $\gamma = 2.360 \pm 0.040$, $R_n = 0.189\pm0.015$ and $R_c = 0.664\pm0.011$ (4.7) on the ratios of the cross-sections for the reactions $K^-p \to \Sigma^-\pi^+$, $K^-p \to \Sigma^+\pi^-$, $K^-p \to \Sigma^0\pi^0$ and $K^-p \to \Lambda^0\pi^0$. We have found that in our approach these experimental constraints are fulfilled within an accuracy better than 6%.

Moreover, we have shown that in our approach between three parameters γ, R_n and R_c only two parameters are independent. Assuming that these are γ and R_n we have expressed R_c in terms of γ and R_n. Using the experimental values for the parameters γ and R_n we have obtained $R_c = 0.626 \pm 0.007$ that agrees with experimental value $R_c = 0.664 \pm 0.011$ within an accuracy better than 6%. Most likely that the obtained agreement of our approach with experimental data on γ, R_n and R_c is a consequence of the $SU(3)_{\mathrm{flavour}}$ singlet-octet nature of the resonances $\Lambda(1405)$, $\Lambda(1800)$ and $\Sigma(1750)$.

One of the consequences of the experimental data (4.7) on the cross-sections for inelastic channels of low-energy K^-p scattering and the $SU(3)_{\mathrm{flavour}}$ singlet-octet nature of the resonances $\Lambda(1405)$, $\Lambda(1800)$ and $\Sigma(1750)$ is a suppression of the contribution of the $\Lambda(1800)$-resonance. Indeed, due to the experimental constraints (4.7) the ratio of the coupling constants of the antisymmetric and symmetric $SU(3)_{\mathrm{flavour}}$ phenomenological B_2BP interactions, $\alpha_2 = f_2/g_2$, turns out to be very close to $-1/3$. Since the coupling constant of the $\Lambda(1800)$-resonance with the $\bar{K}N$ pairs is proportional to $(1 + 3\alpha_2)$, it decouples from the $\bar{K}N$ system for $\alpha_2 = -1/3$.

[10] An expansion in powers of the relative momenta should lead to the corrections of order of powers of α, *i.e.* the term of order $O(\sqrt{kq})$ gives a correction of order $O(\alpha)$ and so on, caused by Coulombic photons. We are planning to analyse these corrections in our forthcoming publications.

For the numerical analysis of the amplitude of K^-p scattering near threshold we have used the recommended values for the masses and total widths of the resonances $\Lambda(1405)$ and $\Sigma(1750)$: $m_{\Lambda(1405)} = 1406\,\mathrm{MeV}$, $\Gamma_{\Lambda(1405)} = 50\,\mathrm{MeV}$ and $m_{\Sigma(1750)} = 1750\,\mathrm{MeV}$ and $\Gamma_{\Sigma(1750)} = 90\,\mathrm{MeV}$. This has given the following value of the resonant part of the amplitude of K^-p scattering near threshold:

$$f_0^{K^-p}(0)_R = (-\,0.154\pm0.009)+i\,(0.269\pm0.032)\ \mathrm{fm}. \quad (7.3)$$

Since the smooth elastic background should be fully real, the imaginary part of $f_0^{K^-p}(0)_R$ coincides with the imaginary part of the S-wave amplitude $f_0^{K^-p}(0)$ of K^-p scattering near threshold. As a result it should fit the experimental data on the width of the energy level of kaonic hydrogen in the ground state. Using the DGBT formula, which is the non-relativistic reduction of our formula (7.1), we have got the value $\Gamma^{\rm th}_{1s} = (227\pm27)$ eV fitting well the mean value of the experimental data by the DEAR Collaboration $\Gamma_{1s} = (213\pm138)$ eV [13].

The shift ϵ_{1s} of the energy level of kaonic hydrogen in the ground state is defined by the S-wave scattering length $a_0^{K^-p}$ of K^-p scattering. In our approach $a_0^{K^-p}$, the real part of the amplitude $f_0^{K^-p}(0)$, is determined by the sum of the contributions of the resonances and a smooth elastic background: $a_0^{K^-p} = \mathcal{R}e\,f_0^{K^-p}(0) = \mathcal{R}e\,f_0^{K^-p}(0)_R + A_B^{K^-p}$.

We have calculated the contribution of the smooth elastic background within the Effective quark model with chiral $U(3)\times U(3)$ symmetry: $A_B^{K^-p} = (-\,0.328\pm0.033)$ fm. This gives the S-wave scattering length $a^{K^-p} = (-0.482\pm0.034)$ fm and the shift of the energy level of the ground state of kaonic hydrogen $\epsilon^{\rm th}_{1s} = (203\pm15)$ eV, which fits well the experimental data $\epsilon^{\rm exp}_{1s} = (183\pm62)$ eV by the DEAR Collaboration [13].

At the hadronic level we have calculated the parameter $A_B^{K^-p}$ in terms of the contribution coming from all hadron exchanges taken at leading order in ChPT, described by Effective Chiral Lagrangians, and scalar mesons $a_0(980)$ and $f_0(980)$ having an exotic $qq\bar{q}\bar{q}$ (or $\bar{K}K$ molecule) structure. Due to the lack of information about the $a_0(980)NN$ and $f_0(980)NN$ coupling constants, the parameter $A_B^{K^-p}$ has been found to be dependent on an arbitrary parameter ξ. Comparing this expression with that obtained at the quark level we have estimated $\xi = 1.2\pm0.1$. Of course, an additional information about the value of ξ can be extracted from the analysis of the contributions of the $a_0(980)$- and $f_0(980)$-mesons to the reactions of the $\bar{K}N$ interaction at transferred momenta of order of 1 GeV.

Thus, in our approach the S-wave amplitude $f_0^{K^-p}(0)$ of K^-p scattering near threshold is equal to

$$f_0^{K^-p}(0) = (-0.482\pm0.034)+i\,(0.269\pm0.032)\ \mathrm{fm}. \quad (7.4)$$

This leads to the following theoretical prediction for the energy level displacement of the ground state of kaonic hydrogen

$$-\epsilon^{\rm th}_{1s} + i\,\frac{\Gamma^{\rm th}_{1s}}{2} = (-203\pm15)+i\,(113\pm14)\,\mathrm{eV}, \quad (7.5)$$

which fits well the experimental data by the DEAR Collaboration [13]

$$-\epsilon^{\rm exp}_{1s} + i\,\frac{\Gamma^{\rm exp}_{1s}}{2} = (-183\pm62)+i\,(106\pm69)\,\mathrm{eV}. \quad (7.6)$$

The calculation of the partial widths of the radiative decay channels of pionic and kaonic hydrogen we have carried out within the soft-pion and soft-kaon technique [48,49][11]. We have shown that for pionic hydrogen the partial width of the decay $A_{\pi p}\to n+\gamma$ gives the Panofsky ratio

$$\frac{1}{P} = \frac{\Gamma(A_{\pi p}\to n\gamma)}{\Gamma(A_{\pi p}\to n\pi^0)} = 0.681\pm0.048 \quad (7.7)$$

agreeing well with the experimental value $1/P = 0.647\pm0.004$ [16].

Unlike pionic hydrogen, where the radiative decay $A_{\pi p}\to n+\gamma$ gives a contribution of about 65%, the contribution of the radiative decay channels $A_{Kp}\to\Lambda^0+\gamma$ and $A_{Kp}\to\Sigma^0+\gamma$ is less than 1%. The theoretical predictions for the sum of the branching ratios of the radiative decay channels of the $\Lambda(1405)$-resonance makes up $(0.13\pm0.03)\%$ [9,62][12].

Thus, the value of the parameter X, supplemented by the contribution of the radiative decays of the $\Lambda(1405)$-resonance, does not exceed 1%. Since both theoretical and experimental accuracy of the definition of the energy level displacement of the ground state of kaonic hydrogen are worse than 1%, one can neglect the contribution of the electromagnetic decay channels of kaonic hydrogen to the total width Γ_{1s}.

Thus, we can argue that strong low-energy $\bar{K}N$ interactions define fully the experimental value of the energy level displacement of kaonic hydrogen measured by the DEAR Collaboration[13].

An agreement of our theoretical predictions for the energy level displacement of the ground state of kaonic hydrogen (7.5) with the experimental data by Iwasaki *et al.* (the KEK experiment) [66]

$$-\epsilon^{\rm exp}_{1s} + i\,\frac{\Gamma^{\rm exp}_{1s}}{2} = (-323\pm63\pm11)+i\,(204\pm104\pm50)\,\mathrm{eV}. \quad (7.8)$$

seems to be only qualitative.

We would like to emphasize that the new data on the energy level displacement have been obtained by the DEAR Collaboration due to a significant improvement of the experimental technique and methodics of the extraction of the energy level displacement of kaonic hydrogen

[11] A constituent-quark diagram technique for the derivation of the soft-pion and soft-kaon low-energy theorems has been elaborated by Natalia Troitskaya in [63] (see also [19,64]).

[12] Theoretical and experimental data on the radiative decays of the $\Sigma(1750)$-resonance are absent [9].

[13] A tangible contribution of about 50% to the paramter X, coming from the isospin-breaking and electromagnetic interactions to the amplitude of low-energy K^-p scattering through the intermediate $\bar{K}^0n$ state $K^-p\to\bar{K}^0n\to K^-p$, has been recently pointed out by Rusetsky [65].

from the data on the $np \to 1s$ transitions, where np is an excited state of kaonic hydrogen [13].

We are grateful to Georgii Shestakov for clarification of exotic properties of the $a_0(980)$- and $f_0(980)$-mesons as the four-quark (or $\bar{K}K$ molecule) states and John Donoghue for discussions. We appreciate discussions with Jürg Gasser and Akaki Rusetsky. We thank Torleif Ericson for the discussions of our approach to the description of the Panofsky ratio for pionic hydrogen and Evgeni Kolomeitsev for the discussions of low-energy theorems for K^-p scattering. The results obtained in this paper have been reported at the Workshop on *CHIRAL DYNAMICS* at University of Bonn, 8-13 September 2003, Germany and at the Workshop on *HADATOM03* at ECT* in Trento, 12-18 October 2003, Italy [14,67].

Appendix A. Calculation of $A_B^{K^-p}$ within Effective quark model with chiral U(3) × U(3) symmetry

Using the expression for the external sources $\eta_p(x_2)$ and $\bar{\eta}_p(x_3)$, given by (5.19), and substituting them in (5.22) we obtain

$$\begin{aligned}&M(K^-p\to K^-p) = i\,\frac{1}{4}\,g^2_B\,g^2_K\,\varepsilon^{i'j'k'}\,\varepsilon^{ijk}\\&\times\int d^4x_1d^4x_2d^4x_3\,e^{i\,q'\cdot x_1+ip'\cdot x_2-ip\cdot x_3}\bar{u}(p',\sigma')_a(i\gamma^5)_{a_1b_1}\\&\times(C\gamma^{\mu})_{a_2b_2}(\gamma_{\mu}\gamma^5)_{ac_2}(\gamma_{\nu}\gamma^5)_{c_3b}(\gamma_{\nu}C)_{a_3b_3}(i\gamma^5)_{a_4b_4}u(p,\sigma)_b\\&\times\langle 0|\mathrm{T}(\bar{u}_{\ell}(x_1)_{a_1}s_{\ell}(x_1)_{b_1}u_{i'}(x_2)_{a_2}u_{j'}(x_2)_{b_2}d_{k'}(x_2)_{c_2}\\&\times\bar{d}_i(x_3)_{c_3}\bar{u}_j(x_3)_{a_3}\bar{u}_k(x_3)_{b_3}\bar{s}_t(0)_{a_4}u_t(0)_{b_4})|0\rangle_c,\qquad (A.1)\end{aligned}$$

where the index c stands for the abbreviation of *connected.*

Making contractions of the d- and s-quark field operators we reduce the r.h.s of (A.1) to the form

$$\begin{aligned}&M(K^-p\to K^-p) = i\,\frac{1}{4}\,g^2_B\,g^2_K\,\varepsilon^{ii'j'}\,\varepsilon^{ijk}\\&\times\int d^4x_1d^4x_2d^4x_3\,e^{i\,q'\cdot x_1+ip'\cdot x_2-ip\cdot x_3}\bar{u}(p',\sigma')_a(i\gamma^5)_{a_1b_1}\\&\times(C\gamma^{\mu})_{a_2b_2}(\gamma_{\mu}\gamma^5)_{ac_2}(\gamma_{\nu}\gamma^5)_{c_3b}(\gamma_{\nu}C)_{a_3b_3}(i\gamma^5)_{a_4b_4}u(p,\sigma)_b\\&\times(-i)\,S^{(s)}_F(x_1)_{b_1a_4}\,(-i)\,S^{(d)}_F(x_2-x_3)_{c_2c_3}\langle 0|\mathrm{T}(\bar{u}_{\ell}(x_1)_{a_1}\\&\times u_{i'}(x_2)_{a_2}u_{j'}(x_2)_{b_2}\bar{u}_j(x_3)_{a_3}\bar{u}_k(x_3)_{b_3}u_{\ell}(0)_{b_4})|0\rangle_c,\quad (A.2)\end{aligned}$$

The requirement to deal with only *connected* quark diagrams prohibits the contraction of the u-quark field operators $\bar{u}_{\ell}(x_1)_{a_1}$ and $u_{\ell}(0)_{b_4}$. The result reads

$$\begin{aligned}&M(K^-p\to K^-p) = 3\,g^2_{\rm B}\,g^2_K\\&\times\int d^4x_1d^4x_2d^4x_3\,e^{i\,q'\cdot x_1+ip'\cdot x_2-ip\cdot x_3}\bar{u}(p',\sigma')_a(\gamma^5)_{a_1b_1}\\&\times(C\gamma^{\mu})_{a_2b_2}(\gamma_{\mu}\gamma^5)_{ac_2}(\gamma_{\nu}\gamma^5)_{c_3b}(\gamma_{\nu}C)_{a_3b_3}(\gamma^5)_{a_4b_4}u(p,\sigma)_b\\&\times S^{(s)}_F(x_1)_{b_1a_4}\,S^{(d)}_F(x_2-x_3)_{c_2c_3}S^{(u)}_F(x_2-x_1)_{a_2a_1}\\&\times S^{(u)}_F(x_2-x_3)_{b_2a_3}S^{(u)}_F(-x_3)_{b_4b_3}.\qquad (A.3)\end{aligned}$$

Summing over the indices we end up with the expression

$$\begin{aligned}&M(K^-p\to K^-p) = 3\,g^2_B\,g^2_K\\&\times\int d^4x_1d^4x_2d^4x_3\,e^{i\,q'\cdot x_1+ip'\cdot x_2-ip\cdot x_3}\bar{u}(p',\sigma')\gamma^{\mu}\gamma^5\\&\times S^{(d)}_F(x_2-x_3)\gamma^{\nu}\gamma^5u(p,\sigma)\mathrm{tr}\Big\{\gamma^5S^{(s)}_F(x_1)\gamma^5S^{(u)}_F(-x_3)\\&\times C^T\gamma^T_{\nu}S^{(u)}_F(x_2-x_3)^T\gamma^T_{\mu}C^TS^{(u)}_F(x_2-x_1)\Big\}.\qquad (A.4)\end{aligned}$$

Using the relation

$$C^T\gamma^T_{\nu}S^{(u)}_F(x_2-x_3)^T\gamma^T_{\mu}C^T = -\gamma_{\nu}S^{(u)}_F(x_3-x_2)\gamma_{\mu}\quad (A.5)$$

we transcribe the r.h.s. of (A.4) into the form

$$\begin{aligned}&M(K^-p\to K^-p) = -\,3\,g^2_B\,g^2_K\\&\times\int d^4x_1d^4x_2d^4x_3\,e^{i\,q'\cdot x_1+ip'\cdot x_2-ip\cdot x_3}\bar{u}(p',\sigma')\gamma^{\mu}\gamma^5\\&\times S^{(d)}_F(x_2-x_3)\gamma^{\nu}\gamma^5u(p,\sigma)\mathrm{tr}\Big\{\gamma^5S^{(s)}_F(x_1)\gamma^5S^{(u)}_F(-x_3)\gamma_{\nu}\\&\times S^{(u)}_F(x_3-x_2)\gamma_{\mu}S^{(u)}_F(x_2-x_1)\Big\}.\qquad (A.6)\end{aligned}$$

In the momentum representation the r.h.s. of (A.6) reads

$$\begin{aligned}&M(K^-p\to K^-p) = 3\,g^2_B\,g^2_K\int\frac{d^4k_1}{(2\pi)^4i}\frac{d^4k_2}{(2\pi)^4i}\,\bar{u}(p',\sigma')\\&\times\gamma^{\mu}\gamma^5\frac{1}{m_d-\hat{k}_1}\gamma^{\nu}\gamma^5u(p,\sigma)\mathrm{tr}\Big\{\gamma^5\frac{1}{m_s-\hat{k}_2}\gamma^5\frac{1}{m_u-\hat{k}_2+\hat{q}}\\&\times\gamma_{\nu}\frac{1}{m_u-\hat{k}_2-\hat{k}_1+\hat{p}+\hat{q}}\gamma_{\mu}\frac{1}{m_u-\hat{k}_2+\hat{q}'}\Big\}.\qquad (A.7)\end{aligned}$$

The result of the calculation of momentum integrals within the procedure accepted in the Effective quark model with chiral $U(3)\times U(3)$ symmetry [17–19] is equal to

$$\begin{aligned}M(K^-p\to K^-p) &= \frac{g^2_B}{8\pi^2}\,\frac{\langle\bar{q}q\rangle}{F^2_K}\,\mu\,\frac{m_s+m}{m_s-m}\\&\times\Big[m^2_s\,\ell n\Big(1+\frac{\Lambda^2_{\chi}}{m^2_s}\Big)-m^2\,\ell n\Big(1+\frac{\Lambda^2_{\chi}}{m^2}\Big)\Big],\quad (A.8)\end{aligned}$$

where $\langle\bar{q}q\rangle = -(252.630\mathrm{MeV})^3$ is the quark condensate, $\Lambda_{\chi} = 940\,\mathrm{MeV}$ is the scale of the spontaneous breaking of chiral symmetry [17,19]. The parameter $A^{K^-p}_B$ is given by

$$\begin{aligned}A^{K^-p}_B &= \frac{M(K^-p\to K^-p)}{8\pi(m_{K^-}+m_p)} = \frac{g^2_B}{64\pi^3}\,\frac{\langle\bar{q}q\rangle}{F^2_K}\,\frac{\mu}{m_{K^-}+m_p}\\&\times\frac{m_s+m}{m_s-m}\Big[m^2_s\,\ell n\Big(1+\frac{\Lambda^2_{\chi}}{m^2_s}\Big)-m^2\,\ell n\Big(1+\frac{\Lambda^2_{\chi}}{m^2}\Big)\Big] =\\&-0.328\,\mathrm{fm}.\qquad (A.9)\end{aligned}$$

A theoretical accuracy of this result is about of 10% [17–19] and [55].

References

1. S. Deser, M.L. Goldberger, K. Baumann, W. Thirring, Phys. Rev. **96**, 774 (1954).
2. K.A. Brueckner, Phys. Rev. **93**, 769 (1955); **107**, 843 (1957); T.L. Trueman, Nucl. Phys. **26**, 57 (1961); A. Deloff, Phys. Rev. C **13**, 730 (1976).
3. A. Deloff, J. Law, Phys. Rev. C **20**, 1597 (1979).
4. A. Deloff, Phys. Rev. C **21**, 1516 (1980).
5. J. Law, M.J. Turner, R.C. Barrett, Phys. Rev. C **35**, 305 (1987).
6. O. Anderson, A.S. Jensen, A. Miranda, G.C. Oades, Phys. Rev. C **41**, R1906 (1990).
7. A.N. Ivanov, M. Faber, A. Hirtl, J. Marton, N.I. Troitskaya, Eur. Phys. J. A **18**, 653 (2003), nucl-th/0306047.
8. A.N. Ivanov, M. Faber, A. Hirtl, J. Marton, N.I. Troitskaya, Eur. Phys. J. A **19**, 413 (2004), nucl-th/0310027.
9. D.E. Groom *et al.*, Eur. Phys. J. C **15**, 1 (2000).
10. The DEAR Collaboration (S. Bianco *et al.*), Riv. Nuovo Cimento **22**, 1 (1999).
11. The DEAR Collaboration (M. Augsburger *et al.*), Nucl. Phys. A **663** & **664**, 561c (2000).
12. The DEAR Collaboration (G. Beer *et al.*), Phys. Lett. B **535**, 52 (2002).
13. The DEAR Collaboration (M. Cargnelli), *DEAR-kaonic hydrogen: First results*, in *Proceedings of the Fourth International Workshop on Chiral Dynamics: Theory and Experiment, Bonn, Germany, 8-13 September, 2003*, p. 125, hep-ph/0311212; invited talk at the *Workshop on HADATOM03, Trento, Italy, 12-18 October, 2003*, hep-ph/0401204.
14. A.N. Ivanov, M. Faber, M. Cargnelli, A. Hirtl, J. Marton, N.I. Troitskaya, J. Zmeskal, *On pionic and kaonic hydrogen*, in *Proceedings of the Fourth International Workshop on Chiral Dynamics: Theory and Experiment, Bonn, Germany, 8-13 September, 2003*, p. 127, hep-ph/0311212.
15. Jürg Gasser, *Comments on Kaon-Nucleon scattering*, in *Proceedings of the Fourth International Workshop on Chiral Dynamics: Theory and Experiment, Bonn, Germany, 8-13 September, 2003*, p. 126, hep-ph/0311212.
16. J. Spuller *et al.*, Phys. Lett. B **67**, 479 (1977).
17. A.N. Ivanov, M. Nagy, N.I. Troitskaya, Phys. Rev. C **59**, 451 (1999)
18. Ya.A. Berdnikov, A.N. Ivanov, V.F. Kosmach, N.I. Troitskaya, Phys. Rev. C **60**, 015201 (1999); A.Ya. Berdnikov, Ya.A. Berdnikov, A.N. Ivanov, V.F. Kosmach, M.D. Scadron, N.I. Troitskaya, Eur. Phys. J. A **9**, 425 (2000); Phys. Rev. D **64**, 014027 (2001); A.Ya. Berdnikov, Ya.A. Berdnikov, A.N. Ivanov, V.A. Ivanova, V.F. Kosmach, M.D. Scadron, N.I. Troitskaya, Eur. Phys. J. A **12**, 341 (2001).
19. A.N. Ivanov, M. Nagy, N.I. Troitskaya, Int. J. Mod. Phys, A **7**, 7305 (1992); Czech. J. Phys. B **42**, 861; 760 (1992); A.N. Ivanov, Phys. Lett. B **275**, 450 (1992); A.N. Ivanov, N.I. Troitskaya, M. Nagy, Phys. Lett. B **295**, 308 (1992); Mod. Phys. Lett. A **7**, 1997, 2095 (1992); Phys. Lett. B **275**, 441 (1992); **308**, 111 (1993); **311**, 291 (1993); Nuovo Cimento A **107**, 1375 (1994); A.N. Ivanov, Int. J. Mod. Phys, A **8**, 853 (1993) A.N. Ivanov, N.I. Troitskaya, M. Nagy, Int. J. Mod. Phys. A **8**, 2027, 3425 (1993); A.N. Ivanov, N.I. Troitskaya, Nuovo Cimento A **08**, 555 (1995); Phys. Lett. B **345**, 175 (1995); **342**, 323 (1995); **387**, 386 (1996); **388**, 869 (1996)(E); **390**, 341 (1997); Nuovo Cimento A **110**, 65 (1997); **111**, 85 (1998); F. Hussain, A.N. Ivanov, N.I. Troitskaya, Phys. Lett. B **348**, 609 (1995); **369**, 351 (1996).
20. S.S. Schweber, in *An Introduction to Relativistic Quantum Field Theory* (Row, Peterson and Co., Evanston, Illinois, Elmsford, New York, 1961).
21. C. Itzykson, J.-B. Zuber, in *Quantum Field Theory* (McGraw-Hill Book Co., New York, 1980).
22. J. Gasser, *Chiral Perturbation Theory*, Nucl. Phys. Proc. Suppl. **86**, 257 (2000); invited talk given at the *High Energy Physics International Euroconference on Quantum Chromo Dynamics - QCD '99, Montpellier, France, 7-13 July 1999*, hep-ph/9912548 and references therein.
23. J. Gasser, H. Leutwyler, Phys. Lett. B **125**, 321; 325 (1983); Ann. Phys. (N.Y.) **158**, 142 (1984); Nucl. Phys. B **250**, 465 (1985); J. Gasser, H. Leutwyler , M.P. Locher, M.E. Sainio, Phys. Lett. B **213**, 85 (1988); J. Gasser, M.E. Sainio, A. Svarc, Nucl. Phys. B **307**, 779 (1988); J. Gasser, H. Leutwyler, M.E. Sainio, Phys. Lett. B **253**, 252; 260 (1991); J. Gasser, M.E. Sainio, *Sigma-Term Physics*, hep-ph/0002283; J. Gasser, V.E. Lyubovitskij, A. Rusetsky, PiN Newslett. **15**, 197 (1999), hep-ph/9911260; J. Gasser, V.E. Lyubovitskij, A. Rusetsky, A. Gall, Phys. Rev. **64**, 016008 (2001).
24. R. Dashen, M. Weinstein, Phys. Rev. **183**, 1261 (1969); **188**, 2330 (1969).
25. J. Gasser, M.A. Ivanov, E. Lipartia, M. Mojžiš, A. Rusetsky, Eur. Phys. J. C **26**, 13 (2002).
26. T.E.O. Ericson, B. Loiseau, S. Wycech, Nucl. Phys. A **721**, 653c (2003), hep-ph/0211433; *Determination of the $\pi^- p$ scattering length from pionic hydrogen*, plenary talk at the *Workshop on HADATOM03 at ECT* in Trento, Italy, 12-18 October 2003*, hep-ph/030134, hep-ph/0401204.
27. T.E.O. Ericson, W. Weise, in *Pions and Nuclei* (Clarendon Press, Oxford, 1988); B. Loiseau, T.E.O. Ericson, A.W. Tomas, PiN Newslett. **15**, 162 (1999), hep-ph/0002056; Nucl. Phys. A **663**, 541 (2000), hep-ph/9907433; **684**, 380 (2001); T.E.O. Ericson, B. Loiseau, A.W. Thomas, Phys. Rev. C **66**, 014005 (2002), hep-ph/0009312.
28. (See [9] pp. 748-750 and p. 120).
29. (See [9] pp. 754-755, pp. 774-775 and p. 120).
30. C.-H. Lee, D.-P. Min, M. Rho, Phys. Lett. B **326**, 14 (1994); C.-H. Lee, G.E. Brown, M. Rho, Phys. Lett. B **335**, 266 (1994); C.-H. Lee, G.E. Brown, D.-P. Min, M. Rho, Nucl. Phys. A **585**, 401 (1995); C.-H. Lee, D.-P. Min, M. Rho, Nucl. Phys. A **602**, 334 (1996).
31. D.N. Tovee *et al.*, Nucl. Phys. B **33**, 493 (1971).
32. R.J. Nowak *et al.*, Nucl. Phys. B **139**, 61 (1978).
33. N. Kaiser, P.B. Siegel, W. Weise, Nucl. Phys. A **594**, 325 (1995).
34. R.H. Dalitz, A. Deloff, J. Phys. G **17**, 289 (1991).
35. V.V. Abaev, B.N.K. Nefkens, Phys. Rev. C **53**, 385 (1996).
36. S. Weinberg, Phys. Rev. Lett. **17**, 616 (1966).
37. B. di Claudio, A.M. Rodriguez-Vargas, G. Violini, Z. Phys. C **3**, 75 (1979).
38. E.E. Kolomeitsev, in *Kaonen in Kernmaterie*, PhD Thesis, 1998; `http://www.physik.tu-dresden.de/publik/1998/diss_kolomeitsev.ps`
39. S. Gasiorowicz, D.A. Geffen, Rev. Mod. Phys. **41**, 531 (1969).
40. R.L. Jaffe, Phys. Rev. D **13**, 267; 281 (1977).

41. N.N. Achasov, S.A. Devyanin, G.N. Shestakov, Sov. J. Nucl. Phys. **32**, 566 (1980); Phys. Lett. B **96**, 168 (1980); **108**, 134 (1982); Z. Phys. C **16**, 55 (1982); Sov. Phys. Usp. **27**, 161 (1984); N.N. Achasov, G.N. Shestakov, Z. Phys. C **41**, 309 (1988); N.N. Achasov, Nucl. Phys. B Proc. Suppl. **21**, 189 (1991); N.N. Achasov, G.N. Shestakov, Sov. Phys. Usp. **34**, 471 (1991); N.N. Achasov, V.V. Gubin, V.I. Shevchenko, Phys. Rev. D **56**, 203 (1997); N.N. Achasov, V.N. Gubin, Phys. Rev. D **56**, 4084 (1997); N.N. Achasov, Phys. Usp. **41**, 1149 (1998), hep-ph/9904223; N.N. Achasov, G.N. Shestakov, Phys. At. Nucl. **62**, 505 (1999); N.N. Achasov, Nucl. Phys. A **675**, 279c (2000) N.N. Achasov, A.V. Kiselev, Phys. Lett. B **534**, 83 (2002); N.N. Achasov, Phys. At. Nucl. **65**, 546 (2002).
42. J. Weinstein, N. Isgur, Phys. Rev. Lett. **48**, 659 (1982); Phys. Rev. D **27**, 588 (1983); **41**, 2236 (1990).
43. F.C. Close, N. Isgur, S. Kumano, Nucl. Phys. B **389**, 513 (1993); N. Brown, F. Close, in *The Second* DAΦNE *Physics Handbook*, edited by L. Maiani, G. Pancheri, N. Paver, Vol. **II** (INFN, Frascati, 1995) p. 649.
44. S. Krewald, R.H. Lemmer, F.P. Sassen, *Lifetime of kaonium*, hep-ph/0307288; F.P. Sassen, S. Krewald, J. Speth, Phys. Rev. D **68**, 036003 (2003); S. Krewald, *Kaonium and meson-exchange models of meson-meson interactions*, invited talk at the *Workshop on HADATOM03 at ECT* in Trento, Italy, 12-18 October, 2003*, hep-ph/0401204.
45. A.N. Ivanov, H. Oberhummer, N.I. Troitskaya, M. Faber, Eur. Phys. J. A **7**, 519 (2000), nucl-th/0006049; **8**, 125 (2000), nucl-th/0006050; **8**, 223 (2000), nucl-th/0006051; A.N. Ivanov, V.A. Ivanova, H. Oberhummer, N.I. Troitskaya, M. Faber, Eur. Phys. J. A **12**, 87 (2001), nucl-th/0108067.
46. M. Primavera, *Results from DAPHNE*, in *Proceedings of the Fourth International Workshop on Chiral Dynamics: Theory and Experiment, Bonn, Germany, 8-13 September, 2003*, p. 22, hep-ph/0311212.
47. M.F.M. Lutz, E.E. Kolomeitsev, Nucl. Phys. A **700**, 193 (2002).
48. S.L. Adler, R. Dashen, in *Current Algebras* (Benjamin, New York, 1968).
49. V. De Alfaro, S. Fubini, G. Furlan, C. Rossetti, in *Currents in Hadron Physics* (North-Holland Publishing Co., Amsterdam, London, American Elsevier Publishing Co., Inc., New York, 1973).
50. J.J. de Swart, Rev. Mod. Phys. **35**, 916 (1963); (see [9] p. 209).
51. G.N. Shestakov, private communication.
52. A.N. Ivanov, M. Nagy, N.I. Troitskaya, M.K. Volkov, Phys. Lett. B **235**, 331 (1990).
53. H.-Ch. Schröder *et al.*, Eur. Phys. J. C **21**, 473 (2001).
54. M.A. Shifman, A.I. Veinshtein, V.I. Zakharov, Nucl. Phys. B **147**, 385; 448; 519 (1979); *Proceedings of the Workshop on Non-Perturbative Methods, Montpellier, 9-13 July, 1985*, edited by S. Narison (World Scientific, 1985).
55. A.N. Ivanov, V.M. Shekhter, Yad. Fiz. **31**, 530 (1980); **32**, 796 (1980).
56. M.M. Nagels *et al.*, Nucl. Phys. B **147**, 189 (1979).
57. (See [21] p. 527).
58. H.L. Anderson, E. Fermi, Phys. Rev. **86**, 794 (1952).
59. J. Bernstein, in *Elementary Particles and Their Currents* (W.H. Freeman and Co., San Francisco and London, 1968) p. 277; R.E. Marshak, Riazuddin, C.P. Ryan, in *Theory of Weak Interactions in Particle Physics* (Wiley-Interscience, A Division of John Wiley & Sons, Inc., New York, 1969) p. 403.
60. T. Hamaie, K. Masutani, M. Arima, Nucl. Phys. A **607**, 363 (1996).
61. D.A. Whitehouse *et al.*, Phys. Rev. Lett. **63**, 1352 (1989).
62. H. Burkhardt, J. Lowe, Phys. Rev. C **44**, 607 (1991).
63. N.I. Troitskaya, in *On Low-Interactions of Low-Lying Hadrons Within Quark Models With Chiral* $U(3) \times U(3)$ *Symmetry*, PhD Thesis, University of St. Petersburg, 1990 (in Russian).
64. A.N. Ivanov, M. Nagy, M.D. Scadron, Phys. Lett. B **273**, 137 (1991).
65. A. Rusetsky, a talk at a working seminar at the *Workshop on HADATOM03 at ECT* in Trento, Italy, 12-18 October, 2003*, hep-ph/0401204.
66. M. Iwasaki *et al.*, Phys. Rev. Lett. **78**, 3067 (1997); (see [9] p. 749).
67. A.N. Ivanov, *Radiative decay channels of pionic and kaonic hydrogen*, plenary talk at the *Workshop on HADATOM03 at ECT* in Trento, Italy, 12-18 October, 2003*, hep-ph/0401204.